# Statistical Analysis of Electromagnetic Structures and Antennas Using the Polynomial Chaos Expansion Method

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***Abstract*— Electromagnetic (EM) structures such as antennas, resonators, are widely used in EM communication systems. It is not uncommon to find that the performance characteristics of these structures are not as expected because of the uncertainties introduced during the manufacturing process. These uncertainties may arise, for instance, from variations of the dimensions of the device or its material properties, e.g., permittivity and permeability. A statistical tool based on the use of the Polynomial Chaos Expansion (PCE) technique is proposed in this work. The results of investigation of two EM structures, namely an E-shaped patch antenna and a split-ring resonator (SRR), are presented as examples. The results confirm that the variability of the design parameters must be taken into account while designing such structures. A sensitivity analysis is also performed in order to determine the most influential parameters that affect the performance of the studied structures. The Sobol indices, whose definitions are provided in this work, are used for this purpose.**

***Index Terms*—antennas, resonator, statistical analysis, polynomial chaos expansion.**

## I. Introduction

Electromagnetic devices such as antennas, resonators, filters, etc., are integral parts of many practical communication systems, radar and bio-electromagnetics [1, 2]. Typically, the manufacturing stage of an electromagnetic device is preceded by a design process during which its performance characteristics, such as gain, directivity, radiation efficiency, resonance frequency, etc., are evaluated. These characteristics depend on many design parameters such as physical dimensions and physical constants. It is not uncommon to find that there are discrepancies between the practical response of a fabricated device and the simulated results for the same that are obtained during the design process. These differences mainly arise due to the uncertainties that might occur in the design parameters during the fabrication process, which in turn affect the performance characteristics or the output quantities mentioned above. As a practical matter, it is often of great interest to estimate the uncertainties in the output responses.

A literature search reveals that there have been numerous studies aimed at estimating the effect of the design parameters on the output response of an EM device [3]. However, these works mainly deal with the individual design parameters, and they only perform a limited number of trials; consequently, they often fail to provide satisfactory results for the desired input parameters. Prominent among these is the Monte Carlo (MC) method, which was first introduced by Fermi for the calculation of neutron diffusion back thein 1930's, and whose aim is to propagate the uncertainties of the input parameters into the output response. Although this technique is reliable for the prediction of the output response distribution of an EM model, it may be very time consuming to implement, since it typically requires us to employ a large number of trials, especially for large problems. Given this background, there exists a great need to develop new powerful techniques for the evaluation of the uncertainties related to EM problems.

In this paper, we focus on the use of a very promising stochastic tool based on the Polynomial Chaos Expansion (PCE) technique that was successfully applied by the authors to the design of a 3D printed GRIN lens in a previous work [4]. In contrast to the MC technique, the convergence in this approach is achieved with only a few simulations. A given number of realization of the input parameters is run through a deterministic code, such as the finite element method that enable ones to perform the PCE analysis as an independent code.

## II. Polynomial Chaos Expansion

The stochastic PCE aims at estimating the uncertainty of the output response of a physical model where its design parameters are considered to be randomly distributed variables. The "non-intrusive" PCE scheme, i.e. the PCE does not modify the deterministic numerical model used for the simulation, is usually preferred. The statistical output response (Y) of the system is computed by propagating the uncertainty of the statistical input set of variables (X) through the approximated meta-model created by the PCE. Assuming that the input parameters are all independent, the polynomial expansion for the constructed meta-model is given by [5]:

$$Y = \sum_{\alpha \in \mathbb{N}^m} a_\alpha \psi_\alpha(X) \qquad (1)$$

In (1) $a_\alpha$ are the unknown coefficients to be determined, $\psi_\alpha(X)$ are the multivariate polynomials and $\alpha$ is the multi-index that identifies the components of $\psi_\alpha(X)$. The unknown coefficients $a_\alpha$ are usually determined by truncating the polynomial expansion, and thus keeping only a subset of polynomials. The Least Angle Regression selection (LARS) is usually preferred for the truncation. In this method, the polynomials that have higher influence on the model output are selected. The coefficients $a_\alpha$ of the truncated expansion are calculated by employing an ordinary least square estimator of type:

$$a = (\Psi^T \Psi)^{-1} \Psi^T y \qquad (2)$$

where $a$ is the vector containing the estimated coefficients, $\Psi$ is a vector of all the polynomials and $y$ is the vector of the output of the model. The aim is to minimize the mean square root error between the output computed by the numerical model and the approximated one given by the truncated PCE. The relative influence of each input parameters on the output of the model is assessed using the Sobol'indices [5].

$$S_i^T = \frac{\sum_{\alpha \in A_i} a_\alpha^2}{\sum_{\alpha \in A[0]} a_\alpha^2} \qquad (3)$$

The denominator of the expression given in (3) is the variance of the output where the mean $a_0$ is excluded while the numerator is the sum of the squared coefficients of the polynomials for the variable $X_i$. For detailed explanation of the PCE and Sobol' indices interested readers are referred to [5].

## III. Application and Results

In this short abstract, only the case examples of two different applications are presented to show the effectiveness of the PCE. The first one is a split ring resonator (SRR) working at THz frequencies (Fig. 1). The input parameters with their nominal values are given in Table 1.

TABLE I. DIMENSIONS OF THE SRR

| ***Input Parameters*** | ***a*** | ***b*** | ***w*** | ***h*** | ***gap*** | ***g*** | ***hs*** |
|---|---|---|---|---|---|---|---|
| ***Nominal Values (μm)*** | 26 | 36 | 4 | 3 | 2 | 2 | 10 |

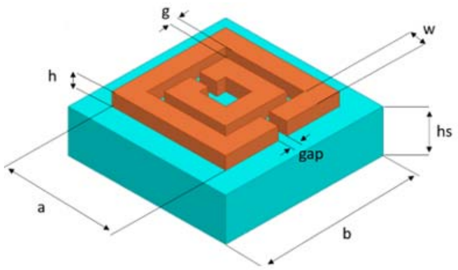


Fig. 1 The Split Ring Resonator on a quartz substrate

The reflectance over the frequency range 0.6-1.8 THz obtained with the PCE analysis along with the MC simulations are presented in Fig. 2. The input parameters are considered to have 5% of uncertainty. The results are compared with those given by the deterministic finite element code where the nominal input values are considered. The results show that the uncertainty of the input parameters induces a relatively high level of variability of the output response, especially around the resonance frequency.

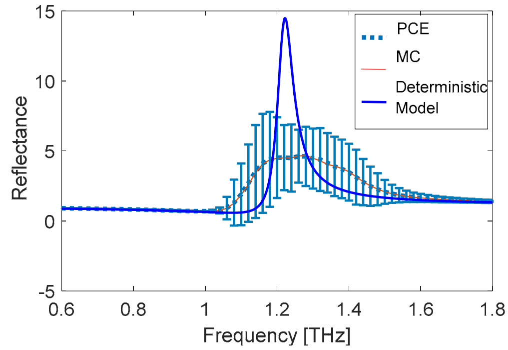


Fig. 2 Output response (reflectance) of the SRR obtained by PCE, MC and the determinitic model

The second example is a dual band E-shaped patch antenna whose operating range is 5-6 GHz, and which is intended for use in wireless commuication systems. The antenna with its input paramters are shown in Fig. 3.

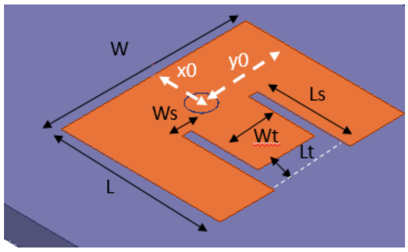


Fig. 3 The E-shaped antenna on a substrate

We present here only the Sobol' indices provided by the PCE analysis. The results presented in Fig. 4 clearly show which of the input parameters has the highest impact on the return loss of the antenna.

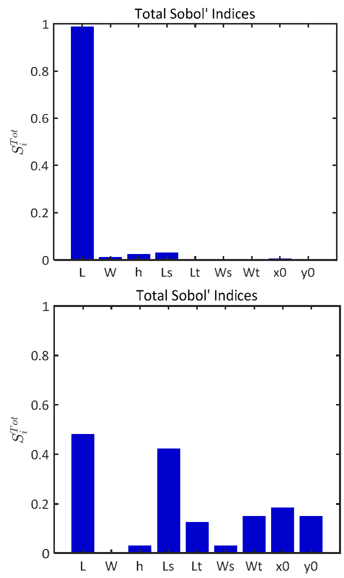


Fig. 4 Sobol' indices at 5.23 GHz (top) and 5.74 GHz (bottom).

At 5.23 GHz the length of the patch is the most influential parameter while at 5.74 GHz the slots dimensions become more influential compared to those at the lower frequency.

## IV. CONCLUSION

The polynomial chaos expansion analysis is increasingly used in different field of engineering. In this paper, we applied it for the sensitivity analysis of some widely used electromagnetic structures. The results show that the output responses of such structures are highly dependent on the variability of the design parameters. Additional examples, together with the associated results will be included in the presentation.

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