

## OPTIMIZATION OF INCLINED BASE CANTILEVER RETAINING WALLS WITH THE HARMONY SEARCH ALGORITHM

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### Abstract

Cantilever retaining walls have been commonly utilized to ensure stability of two different soil levels in geotechnical engineering. In design of cantilever retaining wall, there have been many soil parameters and wall dimensions affecting stability of retaining wall like angle of internal friction, cohesion, wall height, length of base, thickness of base and so on. By using traditional methods, making safety and economic cantilever retaining wall design as soon as possible is time-consuming and trying. Because of these reasons, some effective and easy to apply optimization methods have come into prominence in this kind of design. In this study, optimization of inclined base cantilever retaining wall which is one of the cantilever retaining wall has been investigated with the harmony search algorithm. In the optimum design of the wall, statistically derived mathematical formulas with reasonable absolute relative error have been employed. Safety factors of sliding, overturning and slope stability of the wall have been determined by using these mathematical formulas. These formulas of safety factors are taken as objective function and constraints in process of optimum design. To obtain safety and economic wall design, values of lower and upper limit have been selected for constraints. The length of base, the toe extension, the thickness of base, the slope of base and the angle of internal friction have been taken as design variables with four levels. National and international pre-design guidelines have been used in determination of the lower and upper limits of design variables. For mathematical formulas of all safety factors, 16 inclined base cantilever retaining wall design have been performed by using Taguchi L16 design table which is revised according to five parameters with four level each of them. Obtained results show that harmony search algorithm and mathematical formulas can be used effectively in optimum design of inclined base retaining wall.

**Keywords:** Inclined base cantilever retaining wall; Harmony search algorithm; Optimization

### 1. Introduction

In solution of problem of connecting two different levels each other, retaining structures has been utilized to support the lateral soil loads. Cantilever retaining walls have been commonly used in solution of this kind problem. Performing design of cantilever design as soon as possible is important in term of time and economic criteria. Determination of stability conditions of wall are taken time due to trial-error method which is used in traditional retaining wall design. Initially, safety factors like sliding, overturning and slope stability are calculated according to selected wall dimensions. Because it is repeated until the wall dimensions which satisfy stability conditions of wall are found, this process takes time (Das 2010). Therefore, new methods have been developed by researchers to design and analyze a civil engineering structure in shorter time.

In solution of many engineering problem, optimization methods have been used effectively. Optimization methods are divided as deterministic methods and heuristic methods. Deterministic methods based on mathematical are not enough in situation of being complex engineering problems and equations with multiple variables. Nowadays, heuristic methods which are inspired from the nature have been come into prominence. These algorithms don't guarantee the exact solution but converge to global solution of problem in reasonable time. Harmony search algorithm which is one the heuristic optimization methods has been employed in this study. Harmony search algorithm (HSA) which is first presented by Geem et al. (2001) is based on finding the best harmony in making process of music. HSA has been employed in many engineering problems like structural

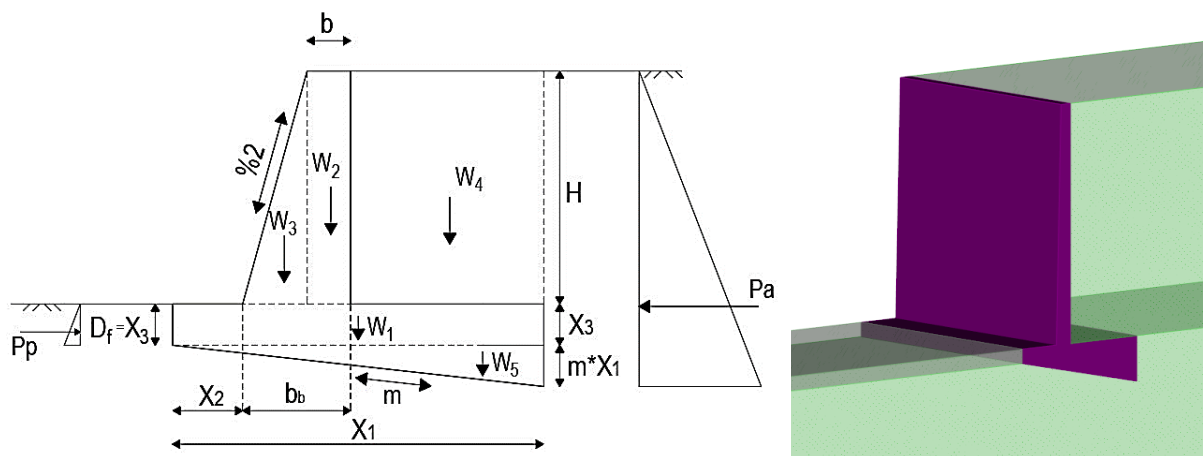
optimization (Çarbaş & Saka 2009), hydraulic (Ayvaz et al. 2013), vehicle routing (Gemm et al. 2005) and geotechnical (Cheng 2009; Khajehzadeh et al. 2011).

In this study, optimization of inclined base cantilever retaining wall which is a kind of retaining wall has been conducted by using HSA. Because, safety factors of sliding, overturning and slope stability have been taken as inequality constraints, stability of retaining wall is ensured in design. Also, the lower and lower limits have been defined for all safety numbers to obtain safe and economic design. By using design parameters which are effect on design of retaining wall have been taken into consideration, the minimum value of safety factors of sliding, overturning and slope stability have been investigated. The length of base, the toe extension, the thickness of base, the slope of base and the angle of internal friction are considered design parameters in the study. Determination of all safety factors for different value of design parameters, mathematical models have been utilized. These models with reasonable relative errors have been developed statistically based on Taguchi method presented by Taguchi (1989).

## 2. Theory

### 2.1. Numerical and statistical analysis

In investigating of stability conditions of inclined base cantilever retaining wall (IBC retaining wall), safety factors of sliding, overturning and slope stability has been taken into consideration. In Figure 1, selected design parameters like the length of base ( $X_1$ ), the toe extension ( $X_2$ ), the thickness of base ( $X_3$ ), the slope of base ( $m$ , %) and the angle of internal friction ( $\emptyset$ , °) are given with acting loads on the wall. In numerical analysis performed by using GEO 5 geotechnical computer program, wall height, top stem thickness of wall, unit volume weight of soil, unit weight of concrete and friction angle between base and soil are taken as respectively  $H=6m$ ,  $b=0.25m$ ,  $\gamma_s=18 \text{ kN/m}^3$ ,  $\gamma_c=25 \text{ kN/m}^3$  and  $\delta=2/3\emptyset$ .



**Figure 1.** Inclined base cantilever retaining wall-GEO5 wall design

Mathematical models of all safety factors have been developed according to selected design parameters and their levels. Selected design parameters and their levels are tabulated in Table 1.

**Table 1.** IBC retaining wall design parameters and their levels

Design parameters	Level 1	Level 2	Level 3	Level 4
$X_1$	0.25H	0.50H	0.75H	1.0H
$X_2$	0.15X1	0.30X1	0.45X1	0.60X1
$X_3$	0.06H	0.09H	0.12H	0.15H
$m$ (%)	8	14	20	26
$\emptyset$ (°)	20	27	34	41

In determination of mathematical models, Taguchi method which is a statistical method has been used (Taguchi, 1950). Taguchi Method is a technique, which gives the results of full factorial study with a small number of

experiment or study by using orthogonal array. This technique is a robust and alternative improved method to determine effects of the parameters on the result. Normally, to investigate effect of five parameters with four levels on safety factors of sliding, overturning and slope stability  $4^5= 1024$  design must be carried out. In this method, it is possible to obtain parameters effect on the result with 16 designs by means of orthogonal array. In this study,  $L_{16}(4^5)$  orthogonal array (five parameters and four level) has been employed and it is given Table 2.

In same table, revised design parameters according to  $L_{16}$  orthogonal array using parameter levels given in Table 1,  $L_{16}$  design table has been revised. Analyses of 16 IBC retaining wall designs has been conducted in computer program according to revised design table and end of the analysis safety factors of sliding, overturning and slope stability have been obtained (Table 3). Safety factor of slope stability has obtained by Bishop (1955) method in computer program.

**Table 2.**  $L_{16}(4^5)$  orthogonal array, IBC retaining wall Taguchi design table and results of numerical analyses

Design No	$L_{16}$ orthogonal array parameter levels					Revised design parameters according to $L_{16}$					Safety factor (Fs) obtained from numerical analyses		
	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	m (%)	Ø (°)	Sliding	Overturning	Slope Stability
1	1	1	1	1	1	0.25H	0.15X <sub>1</sub>	0.06H	8	20	0.30	0.36	0.78
2	1	2	2	2	2	0.25H	0.30X <sub>1</sub>	0.09H	14	27	0.50	0.43	1.15
3	1	3	3	3	3	0.25H	0.45X <sub>1</sub>	0.12H	20	34	0.81	0.51	1.60
4	1	4	4	4	4	0.25H	0.60X <sub>1</sub>	0.15H	26	41	1.49	0.59	2.17
5	2	1	2	3	4	0.50H	0.15X <sub>1</sub>	0.09H	20	41	3.10	3.46	2.46
6	2	2	1	4	3	0.50H	0.30X <sub>1</sub>	0.06H	26	34	1.67	2.65	1.83
7	2	3	4	1	2	0.50H	0.45X <sub>1</sub>	0.15H	8	27	0.72	1.41	1.34
8	2	4	3	2	1	0.50H	0.60X <sub>1</sub>	0.12H	14	20	0.39	0.99	0.92
9	3	1	3	4	2	0.75H	0.15X <sub>1</sub>	0.12H	26	27	1.76	4.74	1.78
10	3	2	4	3	1	0.75H	0.30X <sub>1</sub>	0.15H	20	20	0.91	3.04	1.21
11	3	3	1	2	4	0.75H	0.45X <sub>1</sub>	0.06H	14	41	2.75	6.99	2.38
12	3	4	2	1	3	0.75H	0.60X <sub>1</sub>	0.09H	8	34	1.29	3.90	1.72
13	4	1	4	2	3	1.00H	0.15X <sub>1</sub>	0.15H	14	34	3.38	9.67	2.56
14	4	2	3	1	4	1.00H	0.30X <sub>1</sub>	0.12H	8	41	4.89	12.10	2.87
15	4	3	2	4	1	1.00H	0.45X <sub>1</sub>	0.09H	26	20	1.04	6.91	1.29
16	4	4	1	3	2	1.00H	0.60X <sub>1</sub>	0.06H	20	27	1.20	7.03	1.57

“Signal/Noise Ratio (S/N)” defined by Taguchi with the aim as performance criteria has been utilized for determination of mathematical model. This ratio shows change around the target value and is divided into three according to target; smaller is better, nominal is best, larger is better are given in respectively Equation 1, Equation 2, and Equation 3. In this study, S/N ratios have been determined according to aim of larger is better for 16 IBC retaining wall designs.

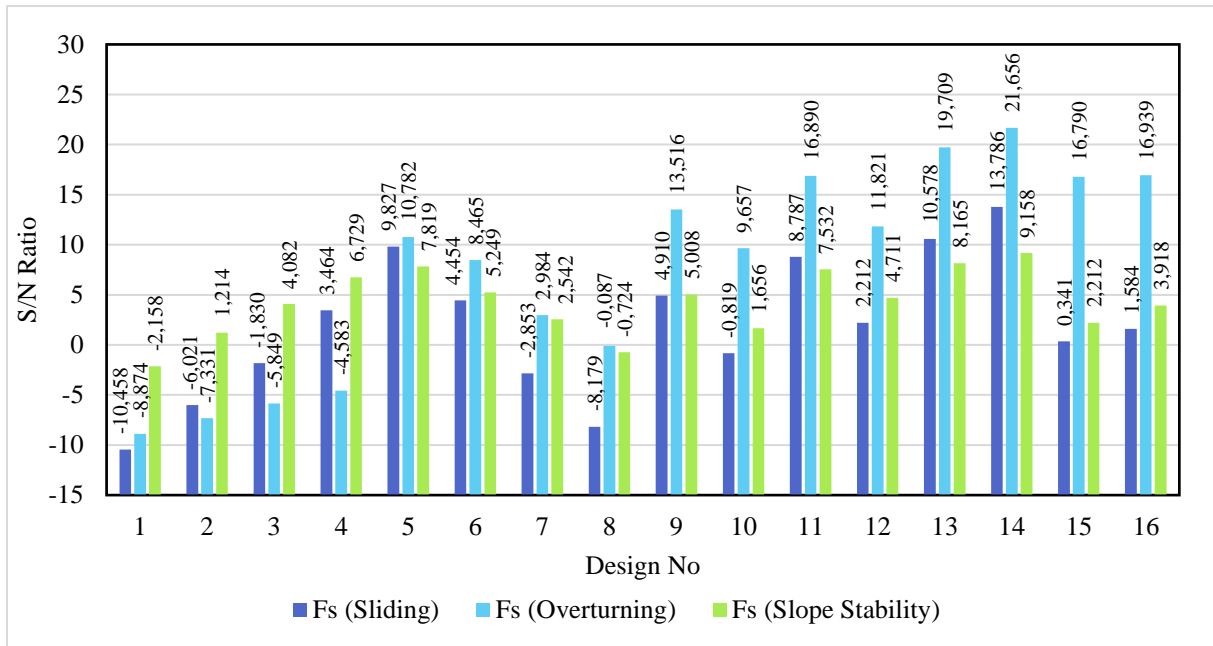
$$S / N = -10 \log(\sum(Y^2) / n) \quad (1)$$

$$S / N = -10 \log(\bar{Y} / \sigma^2) \quad (2)$$

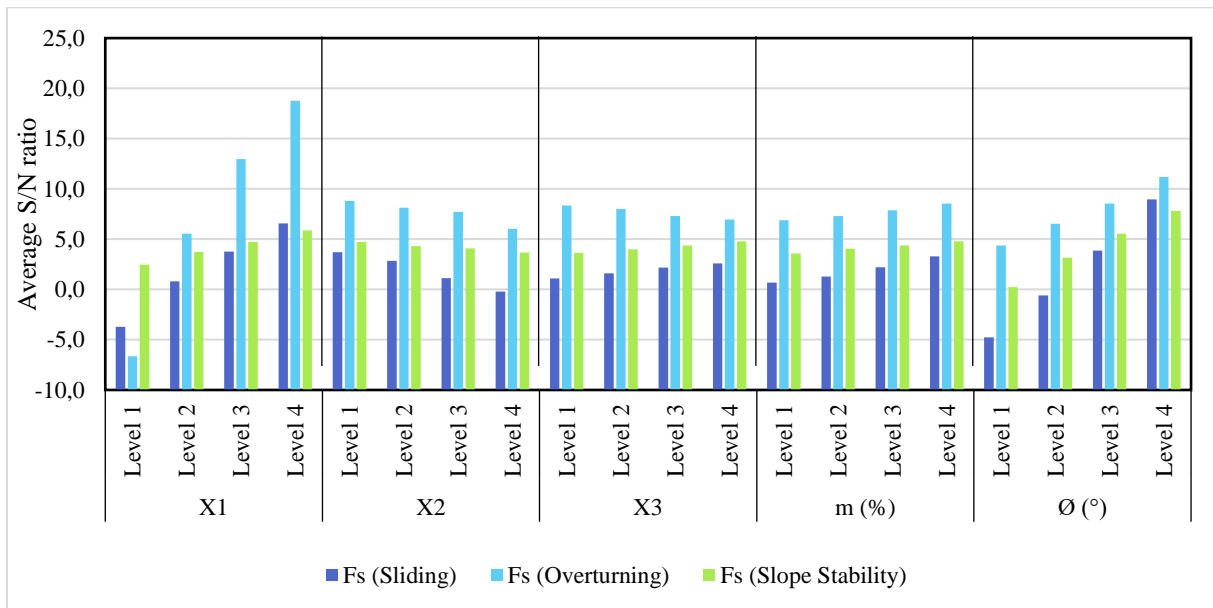
$$S / N = -10 \log(\sum(1 / Y^2) / n) \quad (3)$$

Here Y is the response value, n is the number of repetitions,  $\bar{Y}$  is arithmetic mean and  $\sigma$  is standard deviation.

By using obtained safety factors sliding, overturning and slope stability from numerical analyses, Statistica computer program has been employed for statistical S/N analysis. In Figure 2, calculated S/N ratios by using safety factors obtained from the numerical analyses are given. In Figure 3, change between levels of design parameters and average S/N ratios. of design parameters are presented according to safety factors.



**Figure 2.** S/N ratios for 16 IBC retaining wall designs



**Figure 3.** Average S/N ratios for design parameters

To improve H=6m mathematical model, the average S/N ratios given in Figure 3 have been utilized. Mathematical models valid for given lower-upper levels in Table 1 have been obtained by using average S/N ratios and parameter levels of design parameters. Each of them for calculation of  $F_s$  (sliding),  $F_s$  (overturning) and  $F_s$  (slope stability), mathematical model which is formed using different functions is given by Equation 4.

$$F_m = \sqrt{\frac{1}{10^{-\lambda/10}}} \quad (4)$$

Here,  $\lambda$  is total effect coefficient and it is given by Equation 5.

$$\lambda = \psi_B + \psi_{B_t} + \psi_d + \psi_m + \psi_f + \Delta \quad (5)$$

The detailed description is as follows.

- $\psi_B$  : effect coefficient of the length of base,  $X_1(H)$
- $\psi_{Bt}$  : effect coefficient of the toe extension,  $X_2(X_1)$
- $\psi_d$  : effect coefficient of the thickness of base,  $X_3(H)$
- $\psi_m$  : effect coefficient of the angle of front face,  $m$  (%)
- $\psi_\phi$  : effect coefficient of the angle of internal friction,  $\phi$  ( $^\circ$ )
- $\Delta$  : Coefficient of the average S/N ratio

Value of  $\Delta$  which is changing in terms of calculation of  $F_{ms}$  (safety factor of sliding),  $F_{mo}$  (safety factor of overturning) and  $F_{mss}$  (safety factor of slope stability) are taken as respectively 1.861, 7.655 and 4.195. Detailed explanations of all effect coefficients of parameters are given in Table 9, Table 10 and Table 11 for different safety factors.

**Table 4.** The effect coefficients of parameters of  $F_{ms}$

Lower-Upper Limits of Parameter	Mathematical Model
$0.25 H \leq B \leq 1.00 H$	$\psi_B = 14.969 \left(\frac{B}{H}\right)^3 - 34.962 \left(\frac{B}{H}\right)^2 + 37.767 \left(\frac{B}{H}\right) - 12.6911$
$0.15B \leq B_{\text{ön}} \leq 0.60B$	$\psi_{B_{\text{ön}}} = 62.881 \left(\frac{B_{\text{ön}}}{B}\right)^3 - 76.035 \left(\frac{B_{\text{ön}}}{B}\right)^2 + 18.549 \left(\frac{B_{\text{ön}}}{B}\right) + 0.9416$
$0.06 H \leq d \leq 0.15 H$	$\psi_d = -1516.2 \left(\frac{d}{H}\right)^3 + 456.07 \left(\frac{d}{H}\right)^2 - 25.883 \left(\frac{d}{H}\right) - 0.1587$
$0.08 \leq m \leq 0.26$	$\psi_m = -910.06m^3 + 523.16m^2 - 77.041m + 2.4637$
$20^\circ \leq \phi \leq 41^\circ$	$\psi_\phi = 7.3975(\tan\phi)^3 - 17.181(\tan\phi)^2 + 39.478(\tan\phi) - 18.7171$

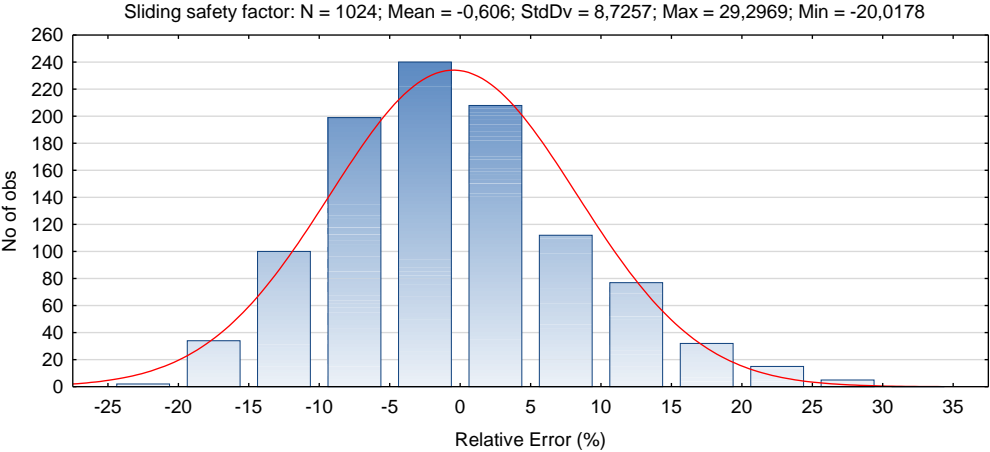
**Table 5.** The effect coefficients of parameters of  $F_{mo}$

Lower-Upper Limits of Parameter	Mathematical Model
$0.25 H \leq B \leq 1.00 H$	$\psi_B = 33.355 \left(\frac{B}{H}\right)^3 - 88.111 \left(\frac{B}{H}\right)^2 + 100.27 \left(\frac{B}{H}\right) - 32.8652$
$0.15B \leq B_{\text{ön}} \leq 0.60B$	$\psi_{B_{\text{ön}}} = -75.849 \left(\frac{B_{\text{ön}}}{B}\right)^3 + 74.107 \left(\frac{B_{\text{ön}}}{B}\right)^2 - 25.875 \left(\frac{B_{\text{ön}}}{B}\right) + 5.1288$
$0.06 H \leq d \leq 0.15 H$	$\psi_d = 4362.5 \left(\frac{d}{H}\right)^3 - 1381.8 \left(\frac{d}{H}\right)^2 - 121.36 \left(\frac{d}{H}\right) - 1.0186$
$0.08 \leq m \leq 0.26$	$\psi_m = -86.502m^3 + 62.603m^2 - 3.9186m + 0.7298$
$20^\circ \leq \phi \leq 41^\circ$	$\psi_\phi = 24.587(\tan\phi)^3 - 46.532(\tan\phi)^2 + 41.255(\tan\phi) - 11.7895$

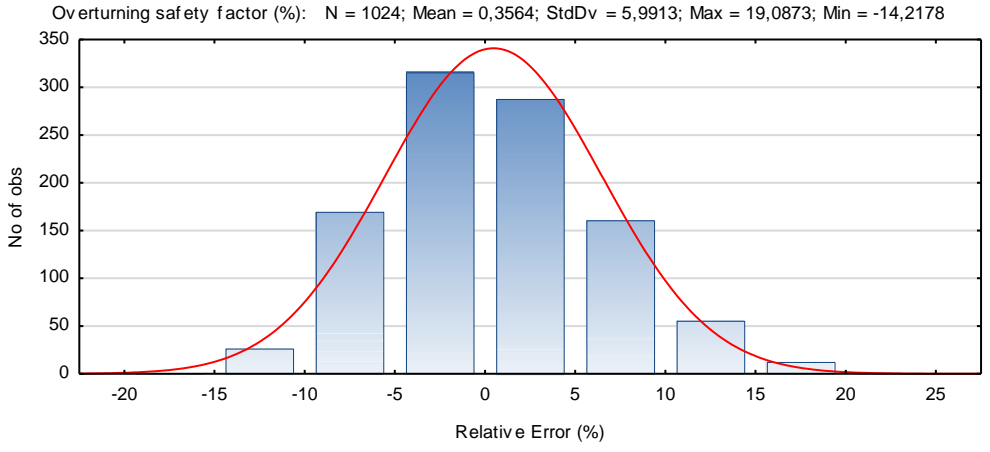
**Table 6.** The effect coefficients of parameters of  $F_{mss}$

Lower-Upper Limits of Parameter	Mathematical Model
$0.25 H \leq B \leq 1.00 H$	$\psi_B = 4.061 \left(\frac{B}{H}\right)^3 - 8.0864 \left(\frac{B}{H}\right)^2 + 9.3063 \left(\frac{B}{H}\right) - 2.7734$
$0.15B \leq B_{\text{ön}} \leq 0.60B$	$\psi_{B_{\text{ön}}} = -18.207 \left(\frac{B_{\text{ön}}}{B}\right)^3 + 19.992 \left(\frac{B_{\text{ön}}}{B}\right)^2 - 8.7245 \left(\frac{B_{\text{ön}}}{B}\right) + 2.2732$
$0.06 H \leq d \leq 0.15 H$	$\psi_d = -241.02 \left(\frac{d}{H}\right)^3 + 86.548 \left(\frac{d}{H}\right)^2 + 2.9274 \left(\frac{d}{H}\right) - 0.1556$
$0.08 \leq m \leq 0.26$	$\psi_m = 208.33m^3 - 109.9m^2 + 24.485m - 1.1547$
$20^\circ \leq \phi \leq 41^\circ$	$\psi_\phi = 20.428(\tan\phi)^3 - 49.844(\tan\phi)^2 + 51.831(\tan\phi) - 16.3566$

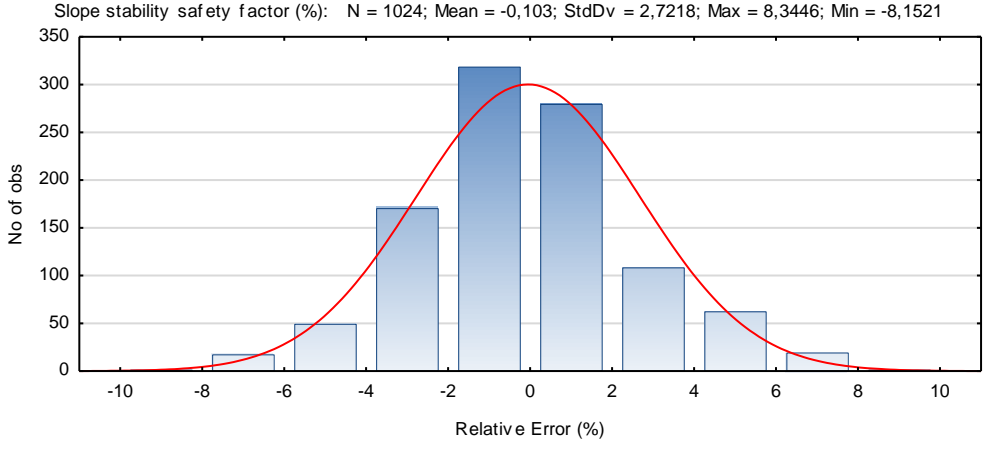
Safety factors of 1024 cantilever retaining wall designs which contain all value of five parameters with four levels have been obtained by both numerical analysis (Fs) and mathematical models (Fm). Belong to safety factors obtained from the numerical analysis and safety factors obtained from mathematical model, the relative error histograms for 1024 safety factors of sliding, overturning and slope stability are given respectively in Figure 4, Figure 5 and Figure 6.



**Figure 4.** Distribution of relative error for safety factor of sliding



**Figure 5.** Distribution of relative error for safety factor of overturning



**Figure 6.** Distribution of relative error for safety factor of slope stability

When histograms of relative error given in figures is examined given figures, it observes that histograms have approximately normal distribution. It is important that data set has a normal distribution in terms of using obtained mathematical model.

## 2.2. Harmony search algorithm

The Harmony search algorithm (HSA) was first presented by Geem et al. (2001). It is a heuristic optimization method based on the principle of finding the best harmony during music performance. HSA is a more advantageous algorithm compared to other heuristic methods because of having a simple algorithm, giving results in reasonable time in cases where iteration number is high, being used for continuous or discrete variables and reaching to global solution in optimization process.

Flowchart of algorithm is given in Figure 7 and detailed explanation of HSA steps are presented below.

*Step 1:* HSA parameters and range for each design parameters are selected. By using these possible range, a design pool is formed. And then harmony memory size (HMS) which is number of solution vectors in harmony memory matrix and also number of rows of harmony memory, harmony memory considering rate (HMCR), pitch adjusting rate (PAR) and maximum number of iterations are determined as HSA parameters.

*Step 2:* The harmony memory matrix (HM) is initialized. The first values are assigned to matrix by selecting randomly from design pool. Each rows of matrix contains possible solutions for related design parameter and these rows are called as solution vectors. HM has N column indicating the number of total design variables. Given in Equation 6,  $X_{i,j}$  means  $i^{\text{th}}$  design variable for  $j^{\text{th}}$  possible solution ( $i=1,2,\dots, n$  and  $j=1,2,\dots,HMS$ ).

$$[H] = \begin{bmatrix} X_{1,1} & X_{2,1} & \dots & X_{n-1,1} & X_{n,1} \\ X_{1,2} & X_{2,2} & \dots & X_{n-1,2} & X_{n,2} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ X_{1,HMS-1} & X_{2,HMS-1} & \dots & X_{n-1,HMS-1} & X_{n,HMS-1} \\ X_{1,HMS} & X_{2,HMS} & \dots & X_{n-1,HMS} & X_{n,HMS} \end{bmatrix} \quad (6)$$

*Step 3:* New harmony matrix is improved. In harmony search method, the creation of a new solution vector is controlled by HMCR and PAR. In process of finding new solution, new solution is selected according to possibility of HMCR from HM. In situation of random selection, design variables of new solution vector are chosen with  $(1-HMCR)$  possibility from design pool. When the design parameter is selected from HM, it is checked whether the selected design parameter is replaced with the lower and upper neighbor of the selected design parameter. This process is called pitch adjusting rate (PAR).

*Step 4:* Harmony memory matrix is updated. Value of objective function is calculated for new solution vector after new solution vector is obtained for each design parameter. If this value is better than the worst value of objection function in the HM, it is included to matrix and the worst value is removed from the matrix.

*Step 5:* HSA is ended if iteration number is reached to maximum iteration number. Until this point Step 3 and Step 4 repeated.

## 3. Optimum Design of Inclined Base Cantilever Retaining Wall

### 3.1. Design parameters

In optimization problem of IBC retaining wall, the length of base ( $X_1$ ), the toe extension ( $X_2$ ), the thickness of base ( $X_3$ ) and the slope of base ( $m$ , %) has been taken as design parameters in Figure 1. The lower and upper limits of optimization design parameters which is included design parameters of mathematical models have been designated in Table 7. Differ from mathematical design parameters the angle of internal friction has been taken as constant and optimization analyses have been performed for varied angle of internal friction ( $20^\circ, 22^\circ, 24^\circ, 26^\circ, 28^\circ, 30^\circ, 32^\circ, 34^\circ, 36^\circ, 38^\circ, 40^\circ$  and  $42^\circ$ ). Design pool has been formed by using value of these design parameters.

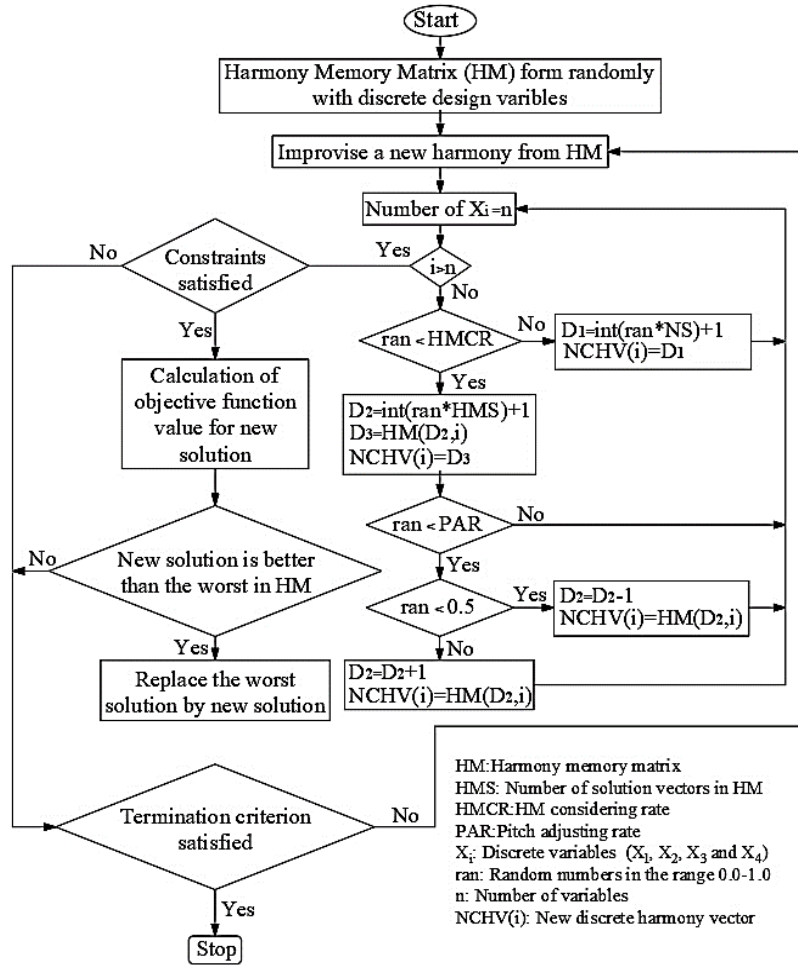


Figure 7. Flowchart of HSA

Table 7. Design parameters of optimization problem

Design parameters	Lower limit	Upper limit	Interval
$X_1$	0.25H	1.0H	0.05H
$X_2$	0.15X1	0.60X1	0.05X1
$X_3$	0.06H	0.15H	0.015H
m (%)	8	26	2

### 3.2. Objective function

Safety factors of sliding, overturning and slope stability have been taken into consideration as objective function of IBC retaining wall optimization problem. Multi objective function which is based on the weighted sum model (Fishburn, 1967 and Triantaphyllou, 2000) has been employed for minimum value of all safety factors in the solution of optimization problem. The percentage by weighted of all safety factors (Fms, Fmo and Fmss) is taken as 0.33. In the solution of the optimization problem, the objective function which is given minimum values of safety factors is given by Equation 7.

$$f_{\min} = 0.33F_{ms} + 0.33F_{mo} + 0.33F_{mss} \quad (7)$$

Fms, Fmo and Fmss correspond to respectively safety factor of sliding, overturning and slope stability given by Equation 4 and Equation 5.

### 3.3. Constraints

IBC retaining wall design, constraints which are given as normalized mathematical expressions are defined by Equation 8 and Equation 9 for safety factor of sliding, are given by Equation 10 and Equation 11 for safety factor



of overturning and are given by Equation 12 and Equation 13 for safety factor of slope stability. In addition, the normalized mathematical expression of the geometric constraints of the wall is given by Equation 14. In this study, the lower limit and upper limits of all safety factors constraints have been chosen respectively 1.3 and 3 to obtain safe and economic wall design.

$$g_x(1) = 1 - \left( F_{ms} / F_{ms_{min}} \right) \leq 0 \quad (8)$$

$$g_x(2) = \left( F_{ms} / F_{ms_{max}} \right) - 1 \leq 0 \quad (9)$$

$$g_x(3) = 1 - \left( F_{mo} / F_{mo_{min}} \right) \leq 0 \quad (10)$$

$$g_x(4) = \left( F_{mo} / F_{mo_{max}} \right) - 1 \leq 0 \quad (11)$$

$$g_x(5) = 1 - \left( F_{mss} / F_{mss_{min}} \right) \leq 0 \quad (12)$$

$$g_x(6) = \left( F_{mss} / F_{mss_{max}} \right) - 1 \leq 0 \quad (13)$$

F<sub>ms</sub>, F<sub>mo</sub> and F<sub>mss</sub> correspond to respectively safety factor of sliding, overturning and slope stability given by Equation 4 and Equation 5. In Table 8, the lower and upper limits of all safety factors (F<sub>ms<sub>min</sub></sub>, F<sub>ms<sub>max</sub></sub>, F<sub>mo<sub>min</sub></sub>, F<sub>mo<sub>max</sub></sub>, F<sub>mss<sub>min</sub></sub> and F<sub>mss<sub>max</sub></sub>) which is given feasible solutions are presented. While safety factors of sliding and slope stability have become 1.3 as the lower limit and 3 as the upper limit for all value of angle of internal friction, safety factor of overturning has same values of limit after Ø=28°. This situation is due to the fact that it does not provide safety factors of constraints at the same design parameters for low values of angle of internal friction.

**Table 8.** The lower-upper limits of safety factors of constraints

Ø (°)	F <sub>ms</sub>		F <sub>mo</sub>		F <sub>mss</sub>	
	Min	Max	Min	Max	Min	Max
20	1.30	3.00	1.30	5.00	1.30	3.00
22	1.30	3.00	1.30	4.50	1.30	3.00
24	1.30	3.00	1.30	4.00	1.30	3.00
26	1.30	3.00	1.30	3.50	1.30	3.00
28-42	1.30	3.00	1.30	3.00	1.30	3.00

### 3.4. Optimum design of IBC retaining wall

In this study, optimum design of IBC retaining wall has been conducted by using Harmony Search Algorithm (HSA). In optimum design problem of wall, the minimum value of objective function has been investigated by using design pool which is created from design parameters given in Table 7. Randomly selected solutions vectors from this design pool have been assigned to harmony memory matrix (HM). For all solutions vector, values of constraints given by Equations 8-14 have been calculated and they have been checked whether they satisfy the lower and upper limits of constraints given by Table 8. For solution vectors which satisfy limits of constraints, values of objective function by using Equation 7 have been determined. According to minimum value of objective function, solutions vector in HM have been sorted from the best value the worst value. In this point, solution vector which has the worst value of objective function removed from HM. This process has been continued, until iteration number has been reached to maximum iteration number.

In this study, HMCR, HMS, PAR and iteration number as HSA parameters are have been selected respectively as 0.95, 30, 0.30 and 10000. Height of IBC retaining wall has been taken as H=6m for different 12 values of angle of internal friction (Ø =20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40 and 42°).

## 4. Results of Optimization Analyses

In optimization of IBC retaining wall, many feasible solutions which satisfy constraints have been gained for all values of angle of internal friction. From among all solutions, the optimum solutions of design parameters which have the lowest minimum objective function value are given in Figure 8. According to figure, optimum results for the length of base (X<sub>1</sub>), the toe extension (X<sub>2</sub>) and the thickness of base (X<sub>3</sub>) have been obtained between

$\phi=30-34^\circ$ . While the most change of the low and upper parameter values of limit happen for  $X_1$ ,  $X_2$  has the least variance in low and upper parameter values.

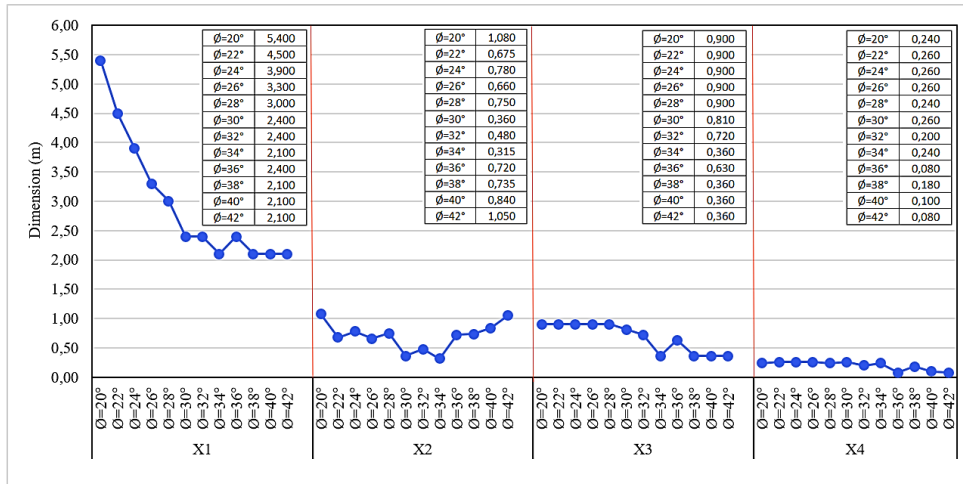


Figure 8. Flowchart of HAS

For values of different angle of internal friction, wall weights and minimum objective function values of optimum IBC retaining wall have been demonstrated in Figure 9. In figure, as the angle of internal friction increases, the weight of wall ( $W_{duvar}$ ) decreases. Value of objective function ( $f_{min}$ ) show decrease until  $\phi=30$  and then stay constant.

According to obtained results of optimum design of IBC retaining wall, change of safety factors of sliding ( $Fms$ ), overturning ( $Fmo$ ) and slope stability ( $Fmss$ ) for different angle of internal friction have been given in Figure 10. By using optimum values given in Figure 8, retaining wall designs have been modelled and analyzed in GEO5 computer program. And then relative errors have been calculated by using safety factors obtained from the optimization analyses ( $Fm$ ) and numerical analyses ( $Fs$ ). In Figure 10, relative errors of all optimum IBC retaining wall designs have been submitted for all safety factors as RE (sliding), Re (overturning) and RE (slope stability). For safety factors of sliding, overturning and slope stability, economic wall designs which safety factors are equal to approximately 1.3 have been obtained after  $\phi=30^\circ$ . When calculated relative errors examine, the maximum absolute relative error is about 10%. It shows that improved mathematical models for safety factors of sliding, overturning and slope stability may be employed effectively and trusting in design of IBC retaining walls.

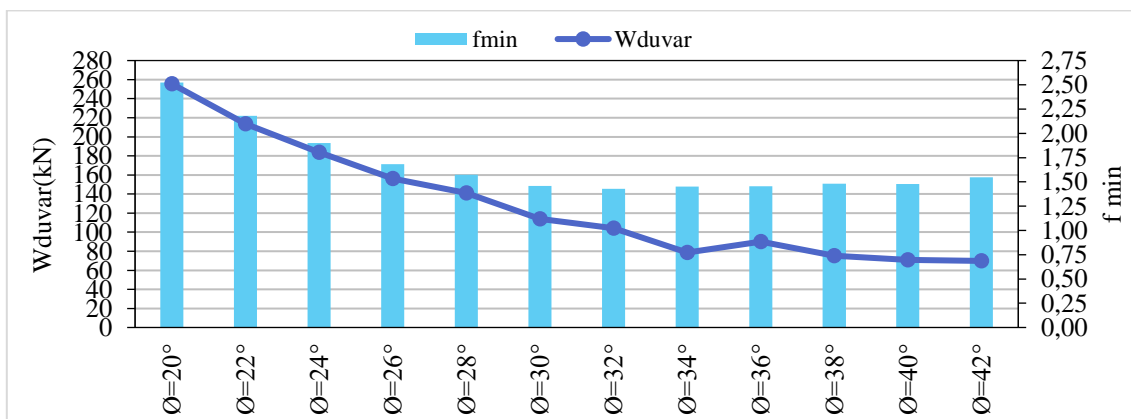
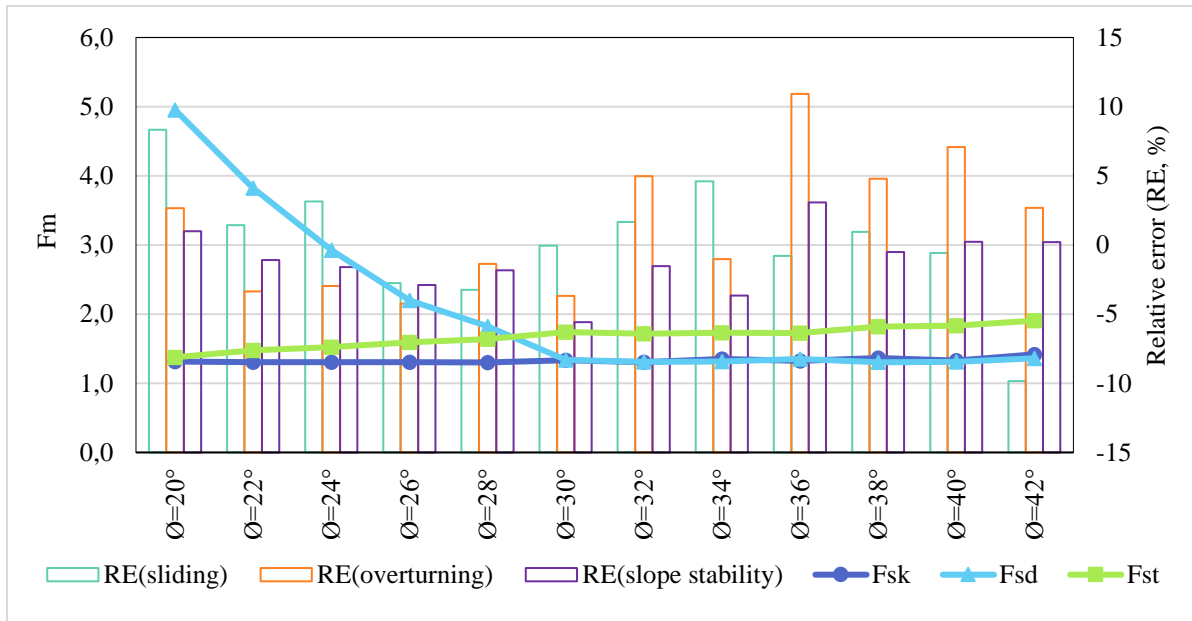


Figure 9. Flowchart of HSA



**Figure 10.** Safety factors and relative errors of optimum wall designs

## 5. Conclusions

In this study, mathematical models for safety factors of sliding, overturning and slope stability have been improved and have been investigated optimization of inclined base cantilever retaining (IBC) wall by using harmony search algorithm (HSA) which is a successful optimization method. Within the scope of the study, the length of base ( $X_1$ ), the toe extension ( $X_2$ ), the thickness of base ( $X_3$ ), the slope of base ( $m$ , %) and the angle of internal friction ( $\emptyset$ , °) have been considered as design parameters for mathematical models of safety factors. In optimization analyses, the angle of internal friction has been taken constant, the others as design parameters. In determination of mathematical models, Taguchi method which is a based statistical method has been employed. According to different levels of design parameters, 16 IBC retaining walls have been modelled and analyzed in GEO5 computer program by means of  $L_{16}$  orthogonal array proposed by Taguchi. In mathematical model of optimization algorithm, developed mathematical models by using Signal/Noise (S/N) ratios of 16 IBC retaining wall have been used as constraints and objective function. Absolute relative errors of 1024 wall designs for safety factors of sliding, overturning and slope stability have been obtained respectively as %6.9, %4.8 and %2.1. This result show that these models can be reliably used in calculation of safety factors of sliding, overturning and slope stability.

In traditional retaining wall design, safety factors of the wall have been determined by using selected wall dimensions. By using trial-error method in design, this process continues until stability of wall satisfies. This method is not only taken time but also it is not guarantee economic design which one is more economical among obtained wall dimensions in design process. Therefore, safety factors ( $F_{ms}$ ,  $F_{mo}$  and  $F_{ms}$ ) have been taken into consideration as constraints and also the lower and upper limits of constraints have been defined respectively as 1.3 and 3.0 to obtain safe and economic design in the optimization analyses.

In optimization analyses, optimum wall design has been investigated for wall height,  $H=6m$  and different angle of internal friction,  $\emptyset = 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40$  and  $42^\circ$ . End of analysis, optimum wall dimensions which satisfy constraints of safety factors and give minimum objective function value have been obtained. Optimum results which safety factors is between 1.3-3.0 have been obtained for  $\emptyset=30^\circ$  and after this value minimum value of objective function stay constant. Because it is impossible having value of safety factors between 1.3-3.0 at the same time for lower angle of internal friction.

Optimum results show that they may be utilized as pre-design guide for defined values of angle of internal friction and  $H=6m$ . Consequently, harmony search algorithm which is a heuristic optimization method can be employed successfully as optimization method in many geotechnical engineering problems.

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