TORSIONAL NATURAL FREQUENCY ANALYSIS OF A CRANKTRAIN SYSTEM USING HOLZER AND FINITE ELEMENT METHOD

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ABSTRACT

This study aims to obtain the torsional natural frequencies and mode shapes of the twelve degrees of freedom cranktrain system consisting of a flywheel, crankshaft, and torsional vibration damper elements using Holzer and finite element methods. An equivalent lumped mass model of the proposed cranktrain system is created by considering twelve masses connected with spring and damping elements. In equivalent modeling approach, eight elements of the lumped mass model are connected in series, the rest four masses are connected by three parallel branchings. Also, these four masses express the torsional vibration damper includes rubber and silicone materials. Generally, the Holzer method is often used to obtain the torsional natural frequency of multi-degree of freedom series-connected systems, therefore a parallel lumped-mass model in which three separate masses are connected to a single mass is not encountered in the literature. Thus, in this study, the Holzer method approach has been developed to determine the torsional natural frequencies of lumped-mass models obtained as multiple parallel branching. Then the modal analysis is realized using the finite element method to compare the obtained results as to the Holzer method. In modal analysis Ansys Workbench software is utilized. While the finite element model is created with 1.2 million degrees of freedom, the Holzer method's model has only twelve degrees of freedom. At the end of the study, the torsional natural frequencies obtained by the Holzer method are compared with the finite element method, and it was determined that it approached 90%.

Keywords: Torsional vibration, Torsional natural frequency, Holzer Method, Modal Analysis, Finite element Method, Ansys Workbench

1. INTRODUCTION

Rotating parts are inherently subject to torsional loads. On the other hand, high-speed rotating components are subjected to torsional vibration and torsional load. Suppose torsional vibration is not included in the design criteria in systems obtained with rotating elements. Even if it is statically safe, the fatigue-life of the system will be low. The most crucial factor that distinguishes torsional vibrations from lateral vibrations is that torsional vibrations cannot be detected until damage occurs. For this reason, the term "insidious vibrations" can be encountered in foreign literature to express the difficulty in detecting torsional vibrations. (Hammady, 2017)

In this study, the cranktrain, an indispensable part of the automotive and machinery sector, was used as a system obtained from rotating elements. The cranktrain system discussed in this study belongs to a four-cylinder high-torque and cycle diesel internal combustion engine. Cranktrain system generally consists of three main parts: flywheel, crankshaft, and torsion vibration pulley. One of the most significant mechanical problems of this type of engine is the crankshaft breakage over dynamic loads. In order to make fatigue life calculations in the frequency or time domain of the crank system, it is necessary to determine the vibration characteristics of the structure, the loading state and the dynamic behaviour of the material used. In this study, the vibration characteristics of cranktrain, a structure exposed to torsional vibration, have been investigated using Holzer and finite element methods.

When the previous studies are examined, the Holzer method has been successfully applied to calculate the natural frequencies of multi-degree freedom rotating systems. For example, (Quiroga et al., 2019) compared the results of a simple three-mass system by calculating the first two natural frequencies using the Holzer and finite element method. The results obtained with the Holzer table obtained from the equivalent model in which three masses were connected in series with a spring approached 98% to the finite element method. (Jayananthan & Shravankumar, 2018) studied the vibration characteristics of the power transmission system of the ship diesel engine. Natural frequency values were determined by the Holzer method from the equivalent model obtained by the discrete mass model. There was a 6.75% difference between the values obtained from the independent design office and the Holzer method. (Tamkhade & Kondhalkar, 2012) designed a viscous torsional damper for a six-cylinder diesel engine. The cranktrain was simplified to obtain an equivalent mass model consisting of ten masses. A Holzer table was constructed using mass moments of inertia, spring constants, and other parameters. By scanning the table with frequency values, it was found that there is a natural frequency where the torque value is equal to zero. This result has shown an accuracy of over 90% by comparing it with experimental methods.

The performance of the modal analysis using the computer-aided finite element method gives satisfactory results compared with the experimental results. (Zhao & Jiang, 2009) calculated the natural frequency of the crankshaft using five different methods, including the finite element method, and compared the results. The finite element method approached a reference value accepted as correct with an error rate of 4.4%. (Shah & Bhabhor, 2014) found the same values with two different computer-aided finite element software Ansys and Hypermesh, to find the crankshaft's natural frequency and mode shapes belonging to a high-speed diesel engine. Mass inertia and stiffness matrices can be obtained very precisely in modal analysis studies using the finite element method. The systems considered can have approximately over one million

degrees of freedom. Therefore, the accuracy of the results obtained is much higher than other numerical methods. Apart from the advantage of this situation, the need for high CPU brings with it the problem of adaptation to different models. Apart from this, when the literature is examined, the software used are commercial products and their prices are very high.
In this study, the Holzer method, which is one of the widely used methods to determine the vibration characteristics of systems exposed to torsional vibration, has been used by modifying it specifically for the system under consideration. The natural frequency values obtained by the modified Holzer method were compared with the results of the computer-aided modal analysis using the finite element method.

2. THE DYNAMIC MODELING OF THE CRANKTRAIN SYSTEM

To dynamically model a real system, a simplified equivalent model is first needed. However, it must be verified that this simplified equivalent model simulates the real system. The cranktrain system discussed in this study has been facilitated by using the lumped mass model. Consisting of twelve masses in total, one of the equivalent model masses is for the flywheel, six for the crankshaft, and five for the torsional vibration damper. The section is shown in green in Fig 1. belongs to the torsional vibration damper. Unlike the other parties, here, the model is obtained using parallel branching. This situation has been acquired by examining both the assembly method and the experimental results.

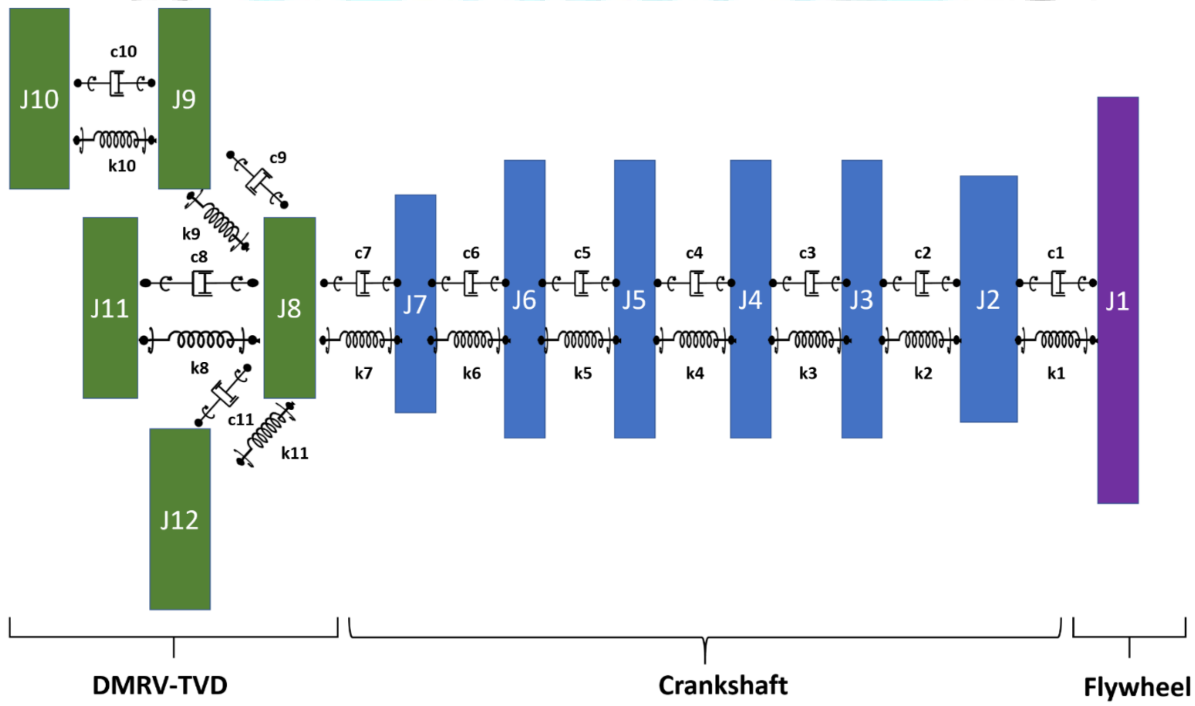


Figure 1. Cranktrain lumped mass model

The motion of equations of a double mass rubber and viscous torsional vibration damper (DMRV-TVD) obtained from the lumped mass model is obtained (Sezgen & Tınkır, 2021). The unknown parameters of the equations of motion, the moment of inertia, stiffness and damping coefficient, are also calculated according to geometry and material properties. Since the equations of motion are created using Newton's second law, they can be adapted to any method in classical mechanics.

2.1. Computer-Aided Finite Element Method: Undamped Free Vibrations of the Cranktrain

Modal analysis, also called frequency analysis, finds natural frequencies and vibration shapes associated with these frequencies. Vibration modes describe a mass that oscillates without damping and forcing forces. While a real-life structure may have an infinite number of degrees of freedom, it still has discrete vibration modes. With its frequency value and associated mode shape, each mode corresponds to the situation where the force due to stiffness is equal to and opposite to the force from inertia (M.Kurowski, 2017).

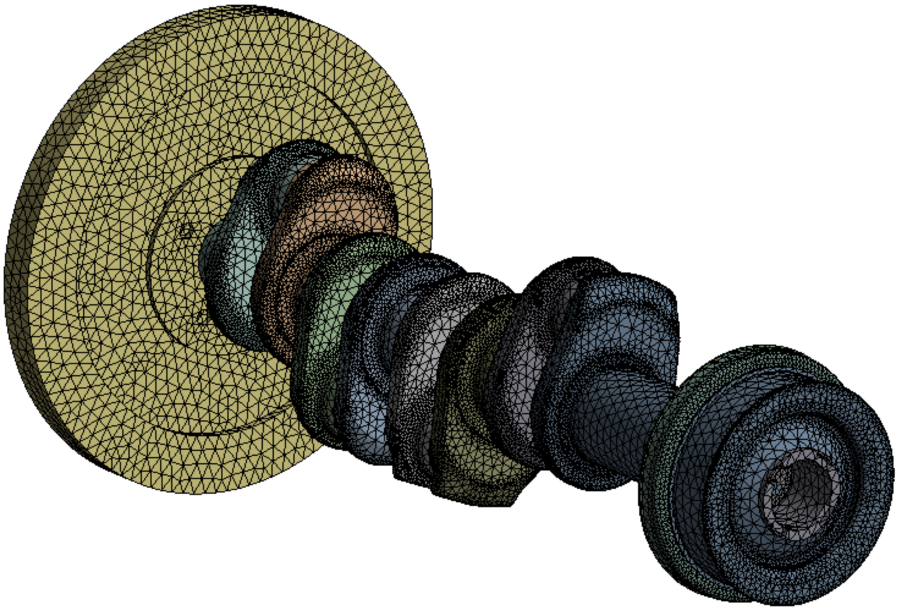


Figure 2. Mesh model of the Cranktrain system

$$[K].[x] = [F] \tag{1}$$
$$[M]\ddot{x} + [C]\dot{x} + [K]x = [F(t)] \tag{2}$$

where
[M]—mass matrix
[C]—damping matrix
[K]—stiffness matrix
[F]—vector of nodal loads
[x]—vector of nodal displacements

Modal analysis deals with free and undamped vibrations where [F(t)] = 0 (no excitation force) and [C] = 0 (no damping). Therefore, Eq. (2) can be simplified to:

$$[M]\ddot{x} + [K]x = 0 \tag{3}$$

Finding nonzero solutions of Eq. (3) presents an eigenvalue problem; it provides modal frequencies and associated mode shapes of vibration:

$$[K]\{\phi\}_j = \omega_i^2[M]\{\phi\}_j \tag{4}$$

Equation Eq. (4) has n solutions, where ω_i^2 is called the eigenvalue, and the corresponding vector $\{\varphi\}i$ is called the eigenvector. The relation between eigenvalue and frequency expressed in Hertz is

$$f_i = \frac{\omega_i}{2\pi} \tag{5}$$

In this study, Ansys software is used to execute the finite element method. Separate material assignment of the parts is made based on a realistic Cad model. Later, connection and boundary conditions are defined to simulate the reality. Another critical issue is determining parametrically a mesh model belonging to the cranktrain system, as shown in Figure 2, to obtain the most accurate result.

2.2. Holzer Method: Undamped Free Vibrations of the Cranktrain

There are methods for finding the natural frequencies of systems such as matrix, finite element, energy and analytical method. The Holzer method is a frequently used method to find torsional natural frequencies and mode shapes of multi-mass rotating elements (Wang, 2013).

The system does not need any external load to maintain its motion while oscillating at the natural frequency value in the undamped free state (Tinkir et al., 2015). In the Holzer method, torsion angle and torque values are calculated for all masses, respectively, with the oscillation value input estimated from one end of the system. The input oscillation value at which the total torque value is equal to zero in the final mass is the system's natural frequency (J.S.Rao, 2011).

The Holzer method is frequently used to find the torsional natural frequencies and mode shapes of rotating elements. However, the classical Holzer method is used only for equivalent models obtained from masses connected in series. For this reason, in this study, a method called the "modified Holzer method" has been developed and made; it can be used for equivalent models obtained from parallel-connected masses.

a

ω hz	ω rad	Moment of inertia Kg.m^2	Unit torque of inertia	Angular amplitude	Inertia torque	Total torque	Stiffness	Twist angle	
		J	Jw²	θ	Jw²θ	ΣJw²θ	ki,i−1	Σ Jw²θ / , -1	
1	6.283185	1.34E-02	0.530542	1.00E+00	5.31E-01	5.31E-01	1.40E+05	3.79E-06	12
1	6.283185	7.28E-05	0.002874	0.999996	2.87E-03	5.33E-01			8
1	6.283185	1.07E-02	0.4224191	1.00E+00	4.22E-01	4.22E-01	1.67E+06	2.53E-07	11
1	6.283185	7.28E-05	0.002874	0.9999997	2.87E-03	4.25E-01	1.18E+06		8
1	6.283185	7.28E-03	0.2873856	1.00E+00	2.87E-01	2.87E-01	1.69E+04	1.70E-05	10
1	6.283185	4.96E-04	1.96E-02	1.0000	1.96E-02	3.07E-01	1.03E+06	2.98E-07	9
1	6.283185	7.28E-05	0.002874	0.99998	2.87E-03	3.10E-01			8
1	6.283185	7.28E-03	0.2873856	2.00003	5.75E-01	5.75E-01	1.69E+04	3.40E-05	10
1	6.283185	4.96E-04	1.96E-02	2.0000	3.92E-02	6.14E-01	1.03E+06	5.96E-07	9
1	6.283185	7.28E-05	0.002874	2.0000	5.75E-03	6.20E-01	1.18E+06	5.25E-07	8
1	6.283185	3.00E-03	1.18E-01	2.0000	2.36E-01	8.56E-01	5.98E+05	1.43E-06	7
1	6.283185	1.50E-02	5.90E-01	2.0000	1.18E+00	2.04E+00	1.73E+06	1.18E-06	6
1	6.283185	1.42E-02	5.61E-01	2.0000	1.12E+00	3.16E+00	1.67E+06	1.89E-06	5
1	6.283185	1.35E-02	5.32E-01	2.0000	1.06E+00	4.22E+00	1.33E+06	3.18E-06	4
1	6.283185	1.27E-02	5.01E-01	2.0000	1.00E+00	5.23E+00	1.10E+06	4.75E-06	3
1	6.283185	3.80E-04	1.50E-02	2.0000	3.00E-02	5.26E+00	7.56E+06	6.95E-07	2
1	6.283185	4.52E-01	1.78E+01	2.0000	3.57E+01	4.09E+01			1
2	12.56637	1.34E-02	2.12E+00	1.00E+00	2.12E+00	2.12E+00	1.40E+05	1.52E-05	12
-									
-									
-									

b

		J	Jw²	θ	Jw²θ	ΣJw²θ	ki,i−1	Σ Jw²θ / , -1	
1	6.283185	1.34E-02	0.530542	1.00E+00	5.31E-01	5.31E-01	1.40E+05	3.79E-06	12
1	6.283185	7.28E-05	0.002874	0.999996	2.87E-03	5.33E-01			8
1	6.283185	7.28E-03	0.2873856	1.00E+00	2.87E-01	2.87E-01	1.69E+04	1.70E-05	10
1	6.283185	4.96E-04	1.96E-02	1.0000	1.96E-02	3.07E-01	1.03E+06	2.98E-07	9
1	6.283185	7.28E-05	0.002874	0.99998	2.87E-03	3.10E-01			8
1	6.283185	1.07E-02	0.4224191	1.00E+00	4.22E-01	4.22E-01	1.67E+06	2.53E-07	11
1	6.283185	7.28E-05	0.002874	0.9999997	2.87E-03	4.25E-01	1.18E+06		8
1	6.283185	1.07E-02	0.4224191	1.99998	8.45E-01	8.45E-01	1.67E+06	5.06E-07	11
1	6.283185	7.28E-05	0.002874	1.9999789	5.75E-03	8.51E-01	1.18E+06	7.21E-07	8
1	6.283185	3.00E-03	1.18E-01	1.9999782	2.36E-01	1.09E+00	5.98E+05	1.82E-06	7
1	6.283185	1.50E-02	5.90E-01	1.9999764	1.18E+00	2.27E+00	1.73E+06	1.31E-06	6
1	6.283185	1.42E-02	5.61E-01	1.9999751	1.12E+00	3.39E+00	1.67E+06	2.03E-06	5
1	6.283185	1.35E-02	5.32E-01	1.9999730	1.06E+00	4.46E+00	1.33E+06	3.35E-06	4
1	6.283185	1.27E-02	5.01E-01	1.9999697	1.00E+00	5.46E+00	1.10E+06	4.96E-06	3
1	6.283185	3.80E-04	1.50E-02	1.9999647	3.00E-02	5.49E+00	7.56E+06	7.26E-07	2
1	6.283185	4.52E-01	1.78E+01	1.9999640	3.57E+01	4.12E+01			1

c

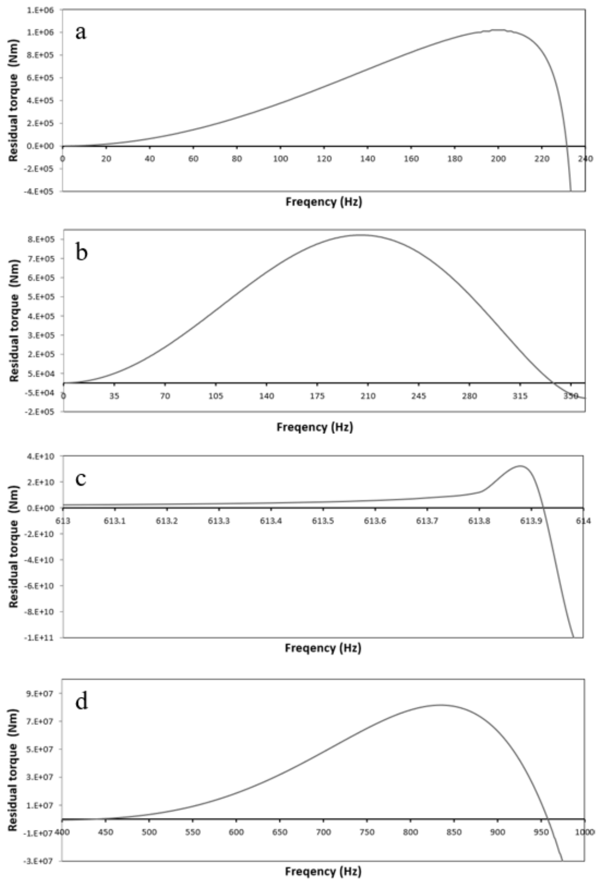
		J	Jw²	θ	Jw²θ	ΣJw²θ	ki,i−1	Σ Jw²θ / , -1	
1	6.283185	7.28E-03	0.2873856	1.00E+00	2.87E-01	2.87E-01	1.69E+04	1.70E-05	10
1	6.283185	4.96E-04	1.96E-02	1.0000	1.96E-02	3.07E-01	1.03E+06	2.98E-07	9
1	6.283185	7.28E-05	0.002874	0.99998	2.87E-03	3.10E-01			8
1	6.283185	1.07E-02	0.4224191	1.00E+00	4.22E-01	4.22E-01	1.67E+06	2.53E-07	11
1	6.283185	7.28E-05	0.002874	0.9999997	2.87E-03	4.25E-01			8
1	6.283185	1.34E-02	0.530542	1.00E+00	5.31E-01	5.31E-01	1.40E+05	3.79E-06	12
1	6.283185	7.28E-05	0.002874	0.999996	2.87E-03	5.33E-01			8
1	6.283185	1.34E-02	0.530542	1.999990	1.06E+00	1.06E+00	1.40E+05	7.58E-06	12
1	6.283185	7.28E-05	0.002874	1.999982	5.75E-03	1.07E+00	1.18E+06	9.04E-07	8
1	6.283185	3.00E-03	1.18E-01	1.999982	2.36E-01	1.30E+00	5.98E+05	2.18E-06	7
1	6.283185	1.50E-02	5.90E-01	1.999979	1.18E+00	2.48E+00	1.73E+06	1.44E-06	6
1	6.283185	1.42E-02	5.61E-01	1.999978	1.12E+00	3.61E+00	1.67E+06	2.16E-06	5
1	6.283185	1.35E-02	5.32E-01	1.999976	1.06E+00	4.67E+00	1.33E+06	3.51E-06	4
1	6.283185	1.27E-02	5.01E-01	1.999972	1.00E+00	5.67E+00	1.10E+06	5.16E-06	3
1	6.283185	3.80E-04	1.50E-02	1.999967	3.00E-02	5.70E+00	7.56E+06	7.54E-07	2
1	6.283185	4.52E-01	1.78E+01	1.999966	3.57E+01	4.14E+01			1

Figure 3. Holzer table for a) First parallel branching, b) Second parallel branching c) Third parallel branching

In this method, the mass and stiffness effects of other branches are transferred to one of the parallel branches, and a solution is made over a single branch. Then the process is repeated for the other parallel branches. As shown in Figure 2, the Holzer table is created as much as the number of parallel branches obtained. The vibration characteristic of the system is obtained by combining the natural frequency values obtained from all these Holzer tables.

3. RESULTS AND DISCUSSION

Frequency scanning is performed from one to the end of the working range in the Holzer tables. Values, where total torque is equal to zero, are considered natural frequency. Accordingly, the frequency - total torque graph is drawn, and the natural frequency values are determined. In Figure 4-a and b, the first and second torsional natural frequency values are seen as 230hz. and 340hz. respectively. In Figure 4-c, the frequency value is not started from zero because the related area becomes obscure as the amplitude diverges excessively. In this graph, it is seen that the third torsional natural frequency value is approximately 614. In Figure 4-d, the fourth natural frequency value in the examined range is found to be 965 hz.



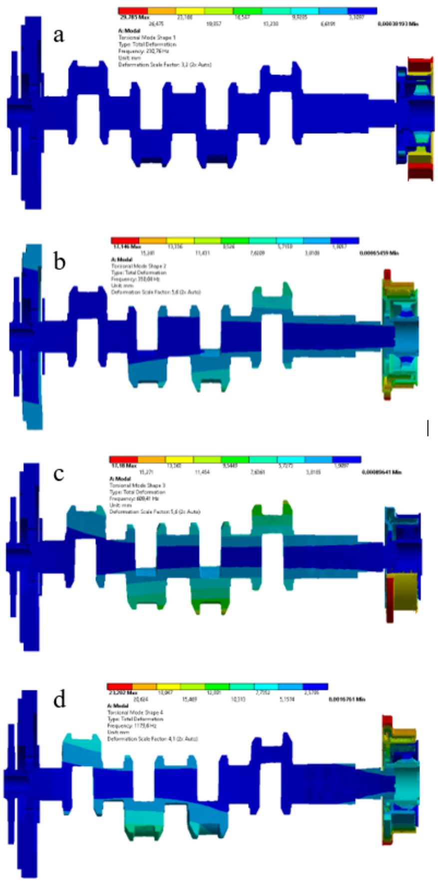


Figure 4 Torsional natural frequency by Holzer method a) first b) second c) third d) fourth

Figure 5 Torsional natural frequency by Finite element method a) first b) second c) third d) fourth

Finite element modal analysis values expected to be at relative values with the Holzer method. Since the torsional vibration damper has parts that are not visible from the outside, a section has been taken as shown in Figure 5 to understand the mode shapes better. In Figure 5-a, the first torsional natural frequency is found on TVD and about 230hz. In Figure 5-b, the second torsional natural frequency value is calculated as 350hz, and the mode shape became effective on the crankshaft. Figure 5-c shows that the third torsional natural frequency is 609 hz. Figure 5-d shows that the frequency value of the last mode shape in the operating frequency range is 1179 hz.

Table 1. Comparative results of the first four torsion natural frequencies using the Finite element and Holzer method.

Method	1st torsional natural frequency	Converge	2nd torsional natural frequency	Converge	3th torsional natural frequency	Converge	4th torsional natural frequency	Converge
Holzer Method	230	99%	340	97%	613,9	99%	965	78%
Finite Element Method	230,76	-	350,04	-	609,41	-	1179,6	-

In Table 1, it is shown how close the Modified Holzer method to the finite element method for the first four natural frequency values. Outside the fourth torsional natural frequency, a satisfactory convergence has been achieved. The divergence of the fourth torsional natural frequency value is that the stiffness and damping coefficients of the materials in the model exhibit viscoelastic behaviour are calculated according to the velocity values in the operating range.

4. CONCLUSIONS

In this study, cranktain, a complex system subjected to torsional vibration, is discussed. The complexity of the system has been studied discontinuously, making it impossible to examine it continuously. Using the computer-aided finite element method, an equivalent model with a mesh structure with over one million degrees of freedom has been obtained. In addition, another equivalent model with twelve degrees of freedom determined by using the discrete system model according to the motion characteristics was obtained. Natural frequency calculations were made by using finite elements with twelve degree of freedom structure and a complex equivalent model using the Holzer method. The results of the Holzer method, which was created and carried out quite simply, were found satisfactory.

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