Statistical Analysis of 3D-Printed Flat GRIN Lenses

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Abstract—**This paper presents the statistical analysis of a 3D printed flat lens by using the Polynomial Chaos Expansion (PCE) analysis technique. The flat lens is fabricated using the 3D printing technology and is based on the grading-index (GRIN) approach. It is composed of several concentric rings with graded relative permittivity, made of a single material with different air holes-host material volume ratio. We show that the hole size can have significant effect on the performance of the lens, especially on the focal distance. PCE analysis enables us also to determine the impact of each individual ring on the performance of the lens.**

Keywords—Polynomial Chaos Expansion; Statistical Analysis; 3D Printed Flat Lens, Grading Index (GRIN) Lens.

I. INTRODUCTION

The propagation of electromagnetic waves and in particular that of the light has been in the scope of scientist for centuries. Lenses have an important place in manipulating such waves. Besides the conventional lenses (curve shaped lenses) that are very well established, flat GRIN lenses have also received great deal of attention. The latter are based on a variation of the refractive index along the radius of the lens. Metamaterials or artificially engineered materials are often used to realize such varying indices. Recently, an all-dielectric flat lens, based on GRIN approach, has been fabricated by means of 3D printing technology [1]. In order to obtain the gradient-index, this new technology enables one to build the flat lens from a single material by introducing holes in the material in a one-step process. By changing the holes/material volume ratio, the desired refractive index or relative permittivity is obtained. Thus, the effective parameters obtained determine the behavior of the lens. However, as for all manufacturing processes, 3D printing technology can only be implemented with some uncertainties in the design parameters, for instance the hole dimensions, thickness of the lens, etc. As we demonstrate in this work, these parameter variabilities may influence the output response of the system. Therefore, the parameters such as the focal length may be affected and the gain may be compromised. In this work, we propose the use of Polynomial Chaos Expansion (PCE) analysis technique [2] to assess the effect of such a design parameter variability on the performance of the lens. Although Monte Carlo (MC) simulations are often used for this purpose, such MC analysis can be very time consuming task, because it requires a large volume of data in order to yield results with high confidence. In contrast to this, PCE analysis has been proven to be an efficient technique that requires relatively low volume of data [2]. Moreover, besides the statistical analysis, PCE analysis enables us to derive important parameters such as probability density function (PDF), the mean and variance values etc., and to determine the influence of each of these design parameters on the output of the lens by using the so-called Sobol' indices. In the following sections, we present the basics of PCE analysis and some early results that have been obtained by using the above analysis.

II. POLYNOMIAL CHAOS EXPANSION

The PCE is a probabilistic method which provides solution to a problem where the model parameters are considered to be randomly distributed variables. It is viewed as a "non-intrusive" method in the sense of that the deterministic numerical model used in the simulation is not modified. By means of a statistical PCE, a meta-model is constructed, then the statistical output response (Y) of the system is computed based on the statistical input set of variables (X). Assuming that the input parameters are all independent, the polynomial expansion for the constructed meta-model is given by:

$$Y = \sum_{\alpha \in \mathbb{N}^m} a_\alpha \psi_\alpha(X) \qquad (1)$$

In (1) a_α are the unknown coefficients to be determined, $\psi_\alpha(X)$ are the multivariate polynomials and α is the multi-index that identifies the components of $\psi_\alpha(X)$. The unknown coefficients a_α are usually determined by truncating the polynomial expansion, and thus retaining only a subset of polynomials. The Least Angle Regression selection (LARS) is usually preferred for the truncation. In this method, the polynomials that have higher influence on the model output are selected. The coefficients a_α of the truncated expansion are calculated by employing a conventional least-square estimator of type:

$$\boldsymbol{\alpha} = (\boldsymbol{\Psi}^T\boldsymbol{\Psi})^{-1}\boldsymbol{\Psi}^T\boldsymbol{y} \qquad (2)$$

where $\boldsymbol{\alpha}$ is the vector containing the estimated coefficients, $\boldsymbol{\Psi}$ is a vector of all the polynomials and $\boldsymbol{y}$ is the vector of the output of the model. The aim is to minimize the mean square root error between the output computed by the numerical model and the approximated one given by the truncated PCE. The relative influence of each input parameters onto the output of the model is assessed by using the Sobol'indices.

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III. APPLICATION AND RESULTS

The 3D printed lens is shown in Fig. 1. It is composed of 6 concentric rings with effective relative permittivity (ε_{reff}) values given in Table 1. The diameter of the lens is 120 mm.

Table 1. Effective relative permittivity values of the 3D printed lens

Ring #	***1***	***2***	***3***	***4***	***5***	***6***
ε_{reff}	2.7	2.42	2.14	1.86	1.58	1.3

The whole 3D lens with sub-millimeter hole dimensions can be time-consuming in terms of simulation. Therefore, it is preferable to use the homogenized lens with the effective relative permittivity values given above. However, in order to determine the effect of the uncertainty of the hole dimensions, a block of the lens composed of 4 holes is considered. Our first objective is to estimate the uncertainty induced by the holes variability on the effective permittivity and then to determine the overall effect on the output response of the 3D lens by using the homogenized lens. The cross-section of the magnified section of the block is shown in Fig. 1.

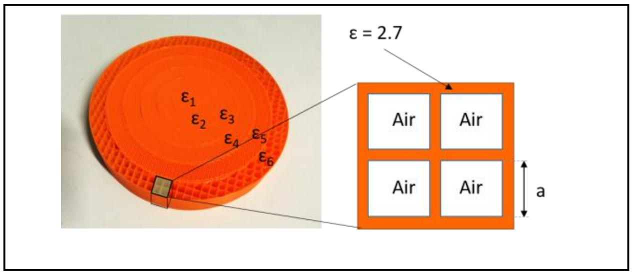

Fig. 1. 3D printed flat lens and the cross section of a block of the outer ring with 4 holes.

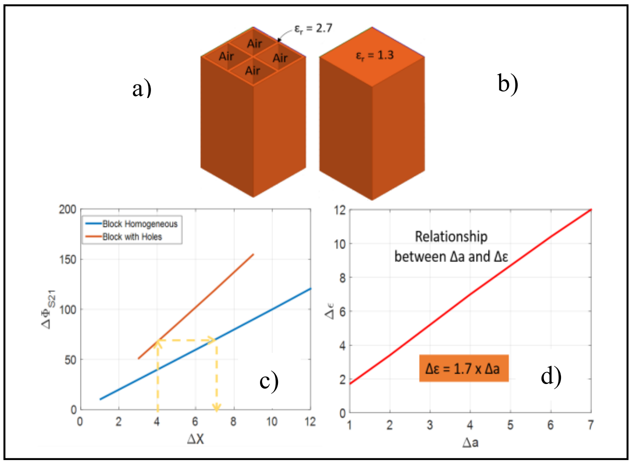

Fig. 2 The procedure for estimating Δε from Δa. a) the block with holes, b) the homogeneous block, c) phase variation for both models, d) the relationship between Δε nd Δa.

In order to estimate the uncertainty induced on the effective relative permittivity, the unit cell simulation is performed using HFSS software. Both the block with holes (Fig. 2a), with ε=2.7 for the material relative permittivity, and the homogeneous block (Fig. 2b) with ε=1.3 as the relative permittivity are simulated. For a certain hole size (or hole/material volume ratio) both structures yield the same electromagnetic response (phase of S_{21}) to an incident plane wave. We introduce some uncertainties in to the hole size (ΔX = Δa for the first case) and in the relative permittivity (ΔX = Δε for the second case) and then compute the variation of the phase of S_{21} (Fig. 2c), to deduce a relationship between Δa and Δε. A linear relation (Δε=1.7Δa) is obtained and is shown in Fig. 2d. In all simulations, the thickness of the 3D lens and that of the block under consideration is assumed to be constant and it is equal to 18.5 mm. The PCE analysis is performed with Δε =8.5%. This corresponds to Δa=5% for the hole size (air) uncertainty.

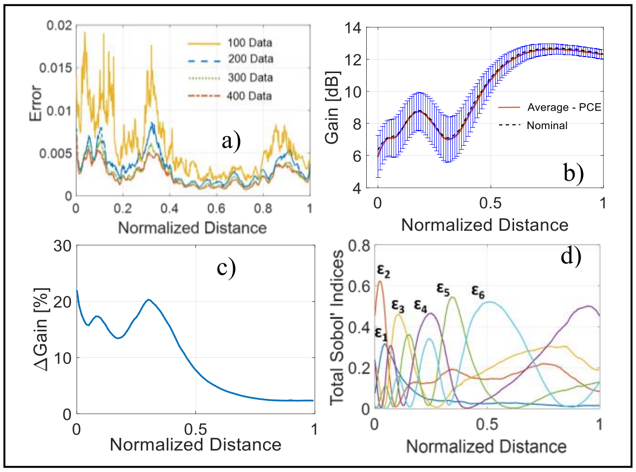

Fig.3. Results obtained by the PCE

Different set of data are considered and the one (400 data) with the lowest error is utilized. The average gain along the axis of the lens, obtained by the PCE and calculated with the nominal values given in Table 1 are plotted in Fig. 3b. The standard deviation of the estimated gain is also represented. It is shown that close to the lens output surface, the induced uncertainty of the gain is higher (Fig. 3c). Finally the relative effect of the effective permittivity of each ring is plotted with respect to the distance to the lens (Fig. 3d).

IV. CONCLUSION

In this paper, a statistical analysis of a 3D printed GRIN lens has been performed using PCE analysis. It has been shown that the variability of the hole size can have a significant effect on the effective permittivity and thus on the output response of the lens.

REFERENCES

[1] S. Zhang, J. C. Vardaxoglou, R. Mittra, "3D Printed Flat Lenses Using Synthetic Artificial Dielectrics", IEEE International Symposium on Antennas and Propagation and North American Radio Science Meeting, 2015.

[2] S. Marelli and B. Sudret, UQLab user manual – Polynomial Chaos Expansions, Report UQLab-V0.9-104, Chair of Risk, Safety & Uncertainty Quantification, ETH Zurich, 2015.