Investigation of Sizing Criteria for Cantilever Retaining Wall with Taguchi Optimization

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***Abstract* - Controllable or uncontrollable parameters like elevation difference between soil levels, soil properties, selected wall dimensions, case of groundwater, construction area, intended use and cost have been affected design of retaining walls that connects two different soil levels. In this study, factors effect on the design of the cantilever retaining walls which have an important place in geotechnical engineering applications have been investigated experimentally. For different wall height and soil properties Signal/Noise (S/N) and variance analysis have been performed separately by using L16 design tables. At result of analysis, effect of the base width, the angle, the toe extension, the footing thickness and the top stem thickness of wall on safety factors of sliding and overturning have been studied. For different conditions of soil and wall, points to consider in pre-sizing of wall have been designated and graphics and tables have been enhanced.**

***Keywords* - Cantilever Retaining Wall, Taguchi Method, Mathematical Model**

I. INTRODUCTION

IN geotechnical engineering, retaining walls are commonly used in the solution of the problem of connecting two different soil levels with a vertical element to provide stability conditions. In the design of retaining wall by using traditional methods, according to selected wall dimensions like the wall height, the length of wall, toe extension, height of stem, thickness of stem and angle of wall necessary verifications have been conducted. Several studies about the design of retaining wall have been made [1, 2]. If the selected wall dimensions do not satisfy verifications like sliding and overturning, according to new selected wall dimensions verifications should have been repeated. This trial and error method continues until stability verifications are provided for selected wall dimensions. This method is not only time consuming but also it does not guarantee that an economic design can be reached. To obtain economic wall dimensions in reasonable time, an optimization method which is based on statistical analysis has been suggested by Taguchi [3]. This method allows to determine the effects of parameters on design and to obtain results with few experiments instead of many experiments [4-6]. The design criteria of the gabion retaining wall, which is another type of retaining wall, has been investigated by using the Taguchi method [7].

In this study, effect of parameters like the length of base, thickness of base, toe extension, thickness of stem and angle of wall on the design of cantilever design has been investigated by Taguchi Method and variance and optimization analysis have been performed. Using the main parameters affecting the design, design tables have been created to be full factorial with five parameters and four levels. All safety factors of sliding and overturning have been determined for each case corresponding to different design. "Signal/Noise, S/N, variance (ANOVA) and optimization analysis have been performed for these safety factors and have been determined effects of parameters on the cantilever retaining wall.

II. CODES USED IN DESIGN OF CANTILER RETAINING WALL

In this section, national and international pre-design guidelines have been given, which enable to determine the appropriate wall dimensions in a shorter time.

In Figure 1, "Soil Retaining Structures; Properties and Guidelines for Design, TS 7994 [8] " is given. Dimensions of the retaining wall firstly, should be determined depend on the wall height. The depth of embedment should be taken as 80 cm for all foundation, but it should be taken as 120cm in frozen region. Toe extension of the retaining wall should be selected B/3 (B: the length of the wall). If verifications do not satisfy, dimensions of l1 and l2 should calculated again

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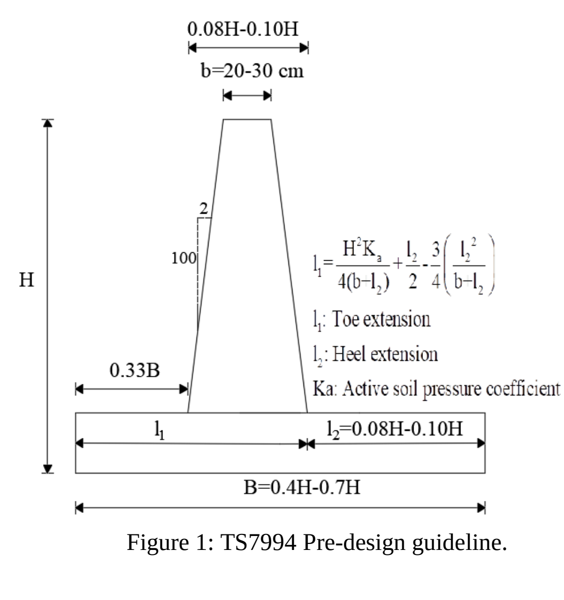

Figure 1: TS7994 Pre-design guideline.

In Figure 2, foreseen wall dimensions according to "Building Code Requirements for Structural Concrete, ACI 318-08 [9] " is given. To obtain economic wall design, the stems of cantilever retaining walls are normally made of constant thickness for wall height up to 12 ft. For above 12 ft heights, the thickness of stem should be made with increasing width bottom to top of stem. The minimum thickness of the base should be selected as 10-12inch [10].

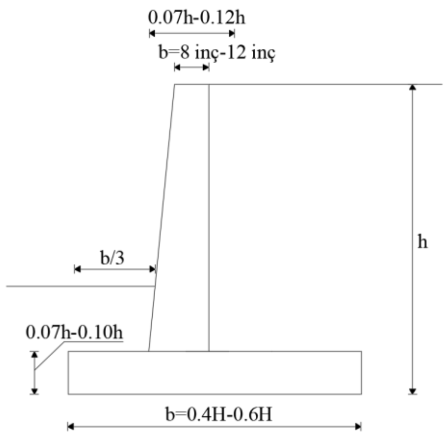

Figure 2: ACI318 Pre-design guideline.

In LRFD Bridge Design Specifications [11], foreseen dimensions of the length of base, thickness of base and toe extension have given. The length of base should be chosen between 70 and 75 percent of the stem height and thickness of base should be chosen between 10 and 15 percent of the stem height. The toe extension should be equal to approximately 30 percent of the length of base.

III. TAGUCHI METHOD

Taguchi is a method which minimizes the effects of uncontrollable factors and limits number of analyses by using orthogonal arrays. In Taguchi Method, result is evaluated by calculating Signal/Noise (S/N) ratio of data obtained from the analysis.

According to Taguchi Method parameters affecting the results are divided into two groups, controllable and uncontrollable parameters. Uncontrollable parameters are density of soil, γ_s= 18 kN/m^3, angle of internal friction, ϕ= 26, 30° and cohesion, c=0. Controllable parameters and their levels which arranged according to orthogonal array levels are given in Table 1.

Table 1: Controllable parameters.

Parameter	Level 1	Level 2	Level 3	Level4
Length of Base (B)	0,25H	0,50 H	0,75 H	1,0 H
Toe Extension (B_t)	0,15 B	0,30 B	0,45 B	0,60 B
Thickness of base (d)	0,05 H	0,07 H	0,09 H	0,11 H
Thickness of stem (b)	0,05 H	0,075 H	0,10 H	0,125 H
Angle of wall (m)	%0	%2	%4	%8

In Taguchi Method, selection of orthogonal array is carried out according to number of level and total degree of freedom. General representation of orthogonal array is $L_{d(a)k}$ or L_d. Here d is the total number of analysis, a is the number of level of parameters, k is number of parameters and L demonstrates orthogonal array. In this study, the number of analysis is 4^5 =1024 and instead of 1024, 16 designs have been made by using L_{16} orthogonal array. In Table 2, controllable parameters which are arranged according to Taguchi are given. According to L16 design table, safety factors of sliding and overturning have been obtained by using computer program for H=4,8 m and Ø=26, 36°.

Table 2: L16 Design Table

No	B	Bt	d	b	m
1	B1	Bt1	d1	b1	m1
2	B1	Bt2	d2	b2	m2
3	B1	Bt3	d3	b3	m3
4	B1	Bt4	d4	b4	m4
5	B2	Bt1	d2	b4	m3
6	B2	Bt2	d1	b3	m4
7	B2	Bt3	d4	b2	m1
8	B2	Bt4	d3	b1	m2
9	B3	Bt1	d3	b2	m4
10	B3	Bt2	d4	b1	m3
11	B3	Bt3	d1	b4	m2
12	B3	Bt4	d2	b3	m1
13	B4	Bt1	d4	b3	m2
14	B4	Bt2	d3	b4	m1
15	B4	Bt3	d2	b1	m4
16	B4	Bt4	d1	b2	m3

IV. SIGNAL/NOISE (S/N), VARIANCE AND OPTIMIZATION ANALYSES

Signal/Noise ratio which is defined by Taguchi is analysis to minimize effect of controllable parameters on result. The S/N ratio, which is a statistical value, is divided according to the target (minimum best, greatest best, and best value target) reached at the end of the study in practice [4]. In this study, calculating of S/N ratio was performed according to the target state of "bigger is better" (1).

$$S/N \ \text{ratio} = -10\log[\sum(1/Y^2)/n] \qquad (1)$$

Where Y is the end of the analysis and n is the number of repetitions. The average S/N ratios for each design have been calculated using the safety factors of sliding and overturning obtained from the computer analysis for different the wall height and the angle of internal friction (Table 3-6).

Table 3: S/N and safety factor values for sliding (Ø=26°).

No	H=4m		H=8m	
	Fs	S/N	Fs	S/N
1	1,322	2,424	1,355	2,638
2	1,293	2,231	1,268	2,062
3	1,264	2,034	1,182	1,452
4	1,174	1,393	0,966	-0,301
5	1,724	4,73	2,193	6,820
6	1,611	4,141	1,903	5,588
7	1,573	3,934	1,772	4,969
8	1,470	3,346	1,504	3,544
9	2,061	6,281	2,892	9,223
10	1,960	5,845	2,603	8,309
11	1,872	5,446	2,343	7,395
12	1,762	4,92	2,034	6,162
13	2,487	7,913	3,782	11,554
14	2,348	7,413	3,387	10,596
15	2,110	6,485	2,780	8,880
16	1,979	5,928	2,403	7,615

Table 4: S/N and safety factor values for sliding (Ø=36°).

No	H=4m		H=8m	
	Fs	S/N	Fs	S/N
1	2,944	9,378	0,619	-4,167
2	2,881	9,190	0,579	-4,747
3	2,819	9,001	0,539	-5,450
4	2,627	8,389	0,437	-7,191
5	3,806	11,609	1,010	0,086
6	3,564	11,038	0,874	-1,170
7	3,481	10,834	0,813	-1,799
8	3,262	10,269	0,688	-3,249
9	4,529	13,120	1,335	2,509
10	4,313	12,695	1,201	1,590
11	4,125	12,308	1,080	0,608
12	3,888	11,794	0,936	-0,575
13	5,444	14,718	1,750	4,860
14	5,145	14,227	1,566	3,895
15	4,635	13,320	1,283	2,164
16	4,354	12,777	1,108	0,890

Table 5: S/N and safety factor values for overturning (Ø=26°).

No	H=4m		H=8m	
	Fs	S/N	Fs	S/N
1	0,846	-1,453	0,848	-1,433
2	0,861	-1,300	0,860	-1,311
3	0,872	-1,190	0,862	-1,290
4	0,727	-2,770	0,623	-4,111
5	2,400	7,604	3,176	10,037
6	2,299	7,230	2,986	9,501
7	2,210	6,887	2,792	8,918
8	1,972	5,898	2,348	7,413
9	4,811	13,644	6,786	16,632
10	4,651	13,350	6,463	16,208
11	4,473	13,011	6,059	15,648
12	4,091	12,236	5,291	14,470
13	8,354	18,437	12,094	22,238
14	8,112	18,182	11,583	21,276
15	7,269	17,229	10,072	20,062
16	6,635	16,436	8,776	18,856

Average S/N ratios corresponding to each parameter level are given in Table 7-10.

Table 6: S/N and safety factor values for overturning (Ø=36°).

No	H=4m		H=8m	
	Fs	S/N	Fs	S/N
1	1,535	3,722	0,542	-5,321
2	1,556	3,840	0,550	-5,193
3	1,574	3,940	0,551	-5,177
4	1,356	2,645	0,392	-8,135
5	3,872	11,758	2,090	6,402

6	3,179	10,045	1,964	5,862
7	3,587	11,094	1,835	5,272
8	3,227	10,175	1,539	3,744
9	7,498	17,498	4,490	13,044
10	7,258	17,216	4,276	12,620
11	6,990	16,889	4,007	12,054
12	6,415	16,143	3,496	10,871
13	12,827	22,162	8,019	18,082
14	12,463	21,912	7,680	17,707
15	11,195	20,980	6,675	16,489
16	10,241	20,206	5,814	15,289

Table 7: Average S/N ratio for sliding safety factor (Ø=26°).

	Parameter	Level 1	Level 2	Level 3	Level 4
H=4m	B	2,021	4,038	5,623	6,935
	Bt	5,337	4,908	4,475	3,897
	d	4,485	4,592	4,769	4,771
	b	4,673	4,734	4,634	4,575
	m	4,525	4,594	4,752	4,746
	Average S/N	4,654			
H=8m	B	1,463	5,23	7,772	9,661
	Bt	7,559	6,639	5,674	4,255
	d	5,809	5,981	6,204	6,133
	b	6,091	6,139	6,049	5,848
	m	5,843	5,967	6,189	6,128
	Average S/N	6,032			

Table 8: Average S/N ratio for sliding safety factor (Ø=36°).

	Parameter	Level 1	Level 2	Level 3	Level 4
H=4m	B	8,990	10,937	12,479	13,761
	Bt	12,206	11,788	11,366	10,807
	d	11,375	11,478	11,654	11,659
	b	11,558	11,621	11,521	11,467
	m	11,416	11,480	11,638	11,633
	Average S/N	11,542			
H=8m	B	-5,389	-1,533	1,048	2,952
	Bt	0,822	-0,108	-1,104	-2,531
	d	-0,945	-0,768	-0,573	-0,635
	b	-0,661	-0,617	-0,721	-0,922
	m	-0,915	-0,787	-0,584	-0,635
	Average S/N	-0,730			

In Figure 3 and 4, graphical representation of average S/N ratio change between S/N and the parameters of length of base and toe extension respectively for safety factor of sliding. In Figure 3, as the length of base increases, safety factor of sliding increases. In Figure 4, as the toe extension increases, safety factor of sliding reduces.

Table 9: Average S/N ratio for overturning safety factor (Ø=26°).

	Parameter	Level 1	Level 2	Level 3	Level 4
H=4m	B	-1,678	6,905	13,061	17,571
	Bt	9,558	9,366	8,984	7,950
	d	8,806	8,942	9,133	8,976
	b	8,963	9,012	9,050	8,833
	m	8,756	8,917	9,178	9,007
	Average S/N	8,965			
H=8m	B	-2,036	8,967	15,739	20,608
	Bt	11,869	11,42	10,834	9,157
	d	10,643	10,82	11,008	10,813
	b	10,808	11	10,953	10,521
	m	10,563	10,77	11,23	10,712
	Average S/N	10,82			

Table 10: Average S/N ratio for overturning safety factor (Ø=36°).

	Parameter	Level 1	Level 2	Level 3	Level 4
H=4m	B	3,536	10,768	16,937	21,315
	Bt	13,785	13,253	13,226	12,292
	d	12,716	13,180	13,381	13,279
	b	13,218	13,267	13,280	12,792
	m	13,023	13,160	13,073	13,301
	Average S/N	13,139			
H=8m	B	-5,957	5,32	12,147	16,892
	Bt	8,052	7,749	7,159	5,442
	d	6,971	7,142	7,329	6,96
	b	7,132	7,172	7,284	6,815
	m	6,883	7,103	7,409	7,007
	Average S/N	7,101			

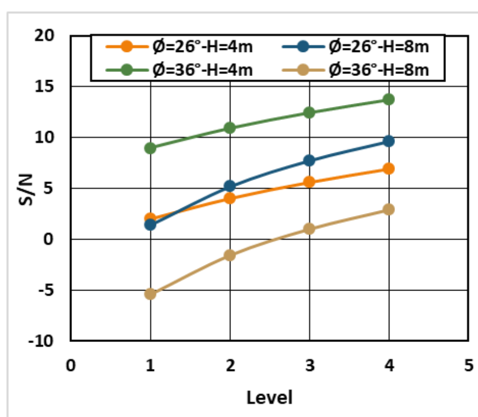

Figure 3: Change between length of base and S/N ratio for safety factor of sliding.

In Figure 5 and 6, graphical representation of average S/N ratio change between S/N and the parameters of length of base and toe extension respectively for safety factor of overturning. In Figure 5, as the length of base increases, safety factor of sliding increases, rapidly. In Figure 6, as toe extension increases, safety factor of sliding shows approximately linear behavior. According to obtained results from S/N analysis, parameters of thickness of base, thickness of stem and wall angle have been determined that there is no effect on the wall design.

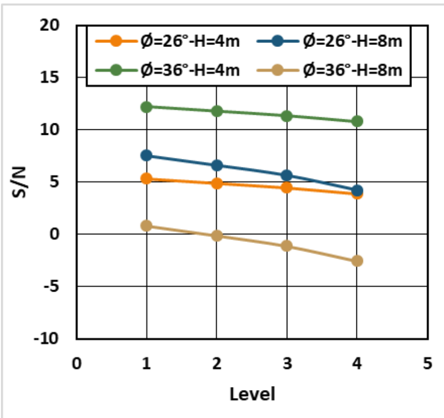

Figure 4: Change between toe extension and S/N ratio for safety factor of sliding.

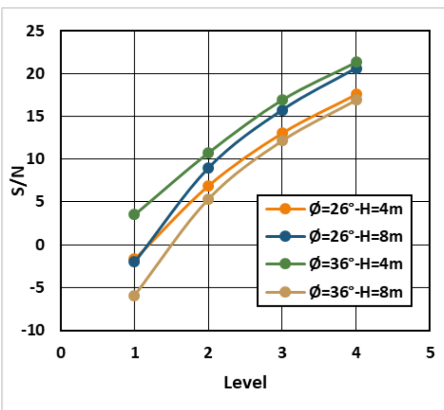

Figure 5: Change between length of base and S/N ratio for safety factor of overturning.

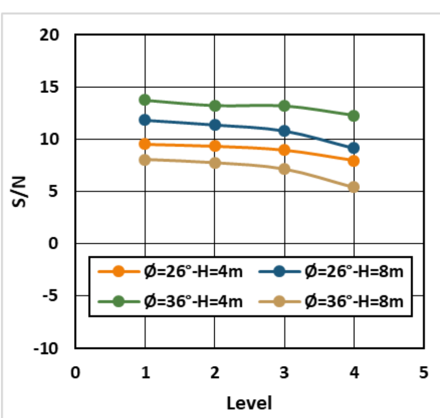

Figure 6: Change between toe extension and S/N ratio for safety factor of overturning.

Variance analysis have been conducted to determine effect of the parameters in the design of cantilever retaining wall and results of analysis are given in Table 11-14.

Table 11: Results of analysis of variance for sliding safety factor (Ø=26°).

H (m)	Parameter	Degree of Freedom (DOF)	Sum of Squares (Ss)	Variance	Percent (P) (%)
4	B	3	53,82	17,94	91,513
	Bt	3	4,546	1,515	7,73
	d	3	0,237	0,079	0,403
	b	3	0,053	0,017	0,09
	m	3	0,153	0,051	0,26
	Total	15	58,809		100
8	B	3	150,881	50,293	85,882
	Bt	3	23,941	7,98	13,627
	d	3	0,367	0,122	0,209
	b	3	0,196	0,65	0,111
	m	3	0,294	0,098	0,167
	Total	15	175,679		100

Table 12: Results of analysis of variance for sliding safety factor (Ø=36°).

H (m)	Parameter	Degree of Freedom (DOF)	Sum of Squares (Ss)	Variance	Percent (P) (%)
4	B	3	50,722	16,907	91,484
	Bt	3	4,289	1,429	7,735
	d	3	0,232	0,077	0,418
	b	3	0,05	0,016	0,091
	m	3	0,149	0,49	0,268
	Total	15	55,442		100
8	B	3	156,276	52,092	85,955
	Bt	3	24,72	8,24	13,596
	d	3	0,324	0,108	0,178
	b	3	0,217	0,072	0,119
	m	3	0,271	0,09	0,149
	Total	15	181,808		100

Table 13: Results of analysis of variance for overturning safety factor (Ø=26°).

H (m)	Parameter	Degree of Freedom (DOF)	Sum of Squares (Ss)	Variance	Percent (P) (%)
4	B	3	883,464	277,821	99,182
	Bt	3	6,17	2,056	0,734
	d	3	0,216	0,072	0,025
	b	3	0,106	0,035	0,012
	m	3	0,373	0,124	0,044
	Total	15	890,329		100
8	B	3	1154,853	384,951	98,406
	Bt	3	16,893	5,631	1,439
	d	3	0,266	0,088	0,022
	b	3	0,553	0,184	0,047
	m	3	0,991	0,33	0,084
	Total	15	1173,556		100

Table 14: Results of analysis of variance for overturning safety factor (Ø=36°).

H (m)	Parameter	Degree of Freedom (DOF)	Sum of Squares (Ss)	Variance	Percent (P) (%)
4	B	3	716,402	238,8	99,102
	Bt	3	4,618	1,536	0,638
	d	3	1,037	0,345	0,143
	b	3	0,65	0,216	0,09
	m	3	0,177	0,059	0,024
	Total	15	722,884		100
8	B	3	1179,957	393,319	98,516
	Bt	3	16,315	5,348	1,362
	d	3	0,362	12	0,03
	b	3	0,484	0,161	0,04
	m	3	0,605	0,201	0,05
	Total	15	1197,723		100

Optimization analysis determines parameter levels and values which make maximum safety factors. These analyses have been applied for safety factors of sliding and overturning for different the wall heights and values of angle of internal friction. As a result of this analysis parameter levels which are given expected maximum safety factors of sliding and overturning have been obtained. According to the obtained parameter levels, safety factors of sliding and overturning have been found by the computer program (Table 15-18).

V. Conclusion

In this study, effects of parameters on design of cantilever retaining wall has been investigated by using Taguchi Method which is one of the strongest optimization technique. Parameters which affect the design of cantilever retaining wall are length of base, toe extension, thickness of base, thickness of stem and wall angle According to Taguchi Method by using L_{16} orthogonal array which consist of four levels and five parameters 16 different designs were modeled and analyzed by computer program to obtain safety factors of sliding and overturning. “S/N, “Variance” and “Optimization” analyses have been performed by using results gained from computer analyses.

S/N analyses show that the safety factors of sliding and overturning has an inverse proportion relationship with increasing of toe extension (Bt) and directly proportional with increasing of length of base (B). The results show that change of values of parameters, thickness of base, thickness of stem and wall angle, does not have any influence on design of cantilever retaining wall.

According to variance analysis, the most effective parameter in design of cantilever retaining wall is the base width with an approximately 90-98 percent and the second effective parameter is the toe extension with an approximately 7 percent.

As a result of optimization analysis generally the maximum safety factors of sliding and overturning have been obtained for B = 1,0H, Bt=0,15B, d =0,09H, b= 0,075H and m=%4.

In this study, prediction and optimization analyses which has been made by using Taguchi Method for safety factors of sliding and overturning, show that results obtained from these analyses is close to real value. Consequently, Taguchi Method can be used in application of geotechnical engineering as an optimization technique.

Table 15: Results of analysis of variance for sliding safety factor (Ø=26°).

H (m)	Parameter	Level	Level Description	Contribution
4	B	4	1,0 H	2,28
	Bt	1	0,15 B	0,683
	d	4	0,11 H	0,117
	b	2	0,075 H	0,079
	m	3	%4	0,098
	Expected maximum safety factor Fs (max) for this level			7,911
	Found by computer analysis maximum safety factor Fs (max)			2,466
8	B	4	1,0 H	3,682
	Bt	1	0,15 H	1,552
	d	3	0,09 H	0,156
	b	2	0,075 H	0,113
	m	3	%4	0,146
	Expected maximum safety factor Fs (max) for this level			4,919
	Found by computer analysis maximum safety factor Fs (max)			1,717

Table 16: Results of analysis of variance for sliding safety factor (Ø=36°).

H (m)	Parameter	Level	Level Description	Contribution
4	B	4	1,0 H	2,218
	Bt	1	0,15 B	0,664
	d	4	0,11 H	0,17
	b	2	0,075 H	0,079
	m	3	%4	0,096
	Expected maximum safety factor Fs (max) for this level			14,716
	Found by computer analysis maximum safety factor Fs (max)			5,397
8	B	4	1,0 H	3,682
	Bt	1	0,15 B	1,552
	d	3	0,09 H	0,156
	b	2	0,075 H	0,113
	m	3	%4	0,146
	Expected maximum safety factor Fs (max) for this level			11,624
	Found by computer analysis maximum safety factor Fs (max)			3,710

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Table 17: Results of analysis of variance for overturning safety factor (Ø=26°).

H (m)	Parameter	Level	Level Description	Contribution
4	B	4	1,0 H	8,606
	Bt	1	0,15 B	0,593
	d	3	0,09 H	0,168
	b	3	0,100 H	0,085
	m	3	4	0,213
	Expected maximum safety factor Fs (max) for this level			18,63
	Found by computer analysis maximum safety factor Fs (max)			8,295
8	B	1,0 H	4	9,79
	Bt	0,15 H	1	0,951
	d	0,09 H	3	0,228
	b	0,10 H	3	0,182
	m	4	3	0,308
	Expected maximum safety factor Fs (max) for this level			18,56
	Found by computer analysis maximum safety factor Fs (max)			7,959

Table 18: Results of analysis of variance for overturning safety factor (Ø=36°).

H (m)	Parameter	Level	Level Description	Contribution
4	B	4	1,0 H	8,175
	Bt	1	0,15 B	0,645
	d	3	0,09 H	0,242
	b	3	0,100 H	0,14
	m	4	8	0,161
	Expected maximum safety factor Fs (max) for this level			22,502
	Found by computer analysis maximum safety factor Fs (max)			12,729
8	B	1,0 H	4	9,79
	Bt	0,15 B	1	0,951
	d	0,09 H	3	0,228
	b	0,075 H	2	0,182
	m	4	3	0,308
	Expected maximum safety factor Fs (max) for this level			22,431
	Found by computer analysis maximum safety factor Fs (max)			11,948

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