Determining of critical safety factors for homogeneous slopes with mathematical model

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ABSTRACT: In this study, the effect of slope and soil parameters on the slope stability has been investigated. The slope heights, the angle of slope, the weight per unit of volume, the angle of internal friction and the cohesion have been used as the slope and the soil parameters. The parameter levels have been selected according to these values and the critic safety factors have been determined with 1280 analyses. In analyses, Slide computer program has been employed and the critic safety factors have been obtained according to Bishop Method. By using these safety factors, the average Signal/Noise (S/N) ratios have been calculated with regard to Taguchi Method for each parameters and the parameter levels. And then statistical analyses have been performed for the selected different mathematical models by using the average S/N ratios. End of study, the best model, which will be used practically in determining of the critic safety factor, has been improved. Consequently, the safety factors calculated with this model have been compared with the safety factors obtained according to the Bishop Method in the computer program for the same parameter levels. It has been observed that the safety factors calculated with the model and the safety factors obtained according to the Bishop Method are close to each other.

1 INTRODUCTION

In slope stability, determining of the critic slide surface is a complex problem of the geotechnical engineering because of many uncertainties. These uncertainties are properties of the soil, the slope parameters, geometry of the slope, existence of ground water, geological history of the soil and so on. To investigate the slope stability, these uncertainties must take into consideration and the minimum safety factor of the slope must be determined properly by making some assumptions. Therefore, several methods such as the Ordinary (Fellenius) (Fellenius 1936), Bishop (Bishop 1955), Janbu (Janbu 1954), Spencer (Spencer 1967), and Morgenstern-Price (Morgenstern and Price 1965) methods have been developed. In these methods, the critical safety factor of slope is generally determined by repeated trials for accepted a failure surface. Several investigations of the slope stability have been reported in literature. Malkawi et al. (2000) has reported a comparison of four popular methods, the Ordinary, Bishop, Janbu and Spencer methods. Nguyen (1985) has developed an optimization technique in order to determine the critical safety factor. Baker (1980) has studied in determination of an approximate critical failure surface by using the Spencer method. Dodagoudar and Venkatachalam (2000) have used the fuzzy sets theory in a slope stability analysis. Leshchinsky (1990) developed a method for the slope stability analysis based on the limit equilibrium. Lu and Rosenbaum (2003) have used artificial neural networks to determine the slope stability. A number of applications of the slope stability studies using numerical approaches have been reported (Cheng 2003) (Combie and Wilkinson 2002). In addition, parameters affecting the stability of homogeneous slopes have been investigated by using Taguchi Method (Tan, 2006). Harmony Search Algorithm, which is a heuristic optimization method, has been used to determine the critic slide face in the slope (Cheng 2009). Analysis of stability of three-dimensional slopes has been studied by using the rigorous limit equilibrium method (Zhou 2013). Bai (2014) has presented a new method of analyzing the slope stability. Gandomi (2015) has used recent swarm intelligence techniques in the slope stability analysis. Mukhlisin (2016) has been investigated effect of water pressure in slope and has presented use of horizontal drains to control ground water in slope stabilization. Analysis of rock slope stability has been studied by using Neural Network and Steepest Descent Algorithm (Li 2016).

In this study, the critic safety factor of homogenous slope has been determined by using the Bishop method. The Bishop (1955) method is a slices method of the slope stability analysis. In this method, while the normal forces are taken in to consideration, the shear forces between the slices are neglected. Calculation of the safety factor has been conducted according to moment equilibrium in a slice. The stresses and forces, which act on a slice, are used to determine the safety factor are given in Figure 1.

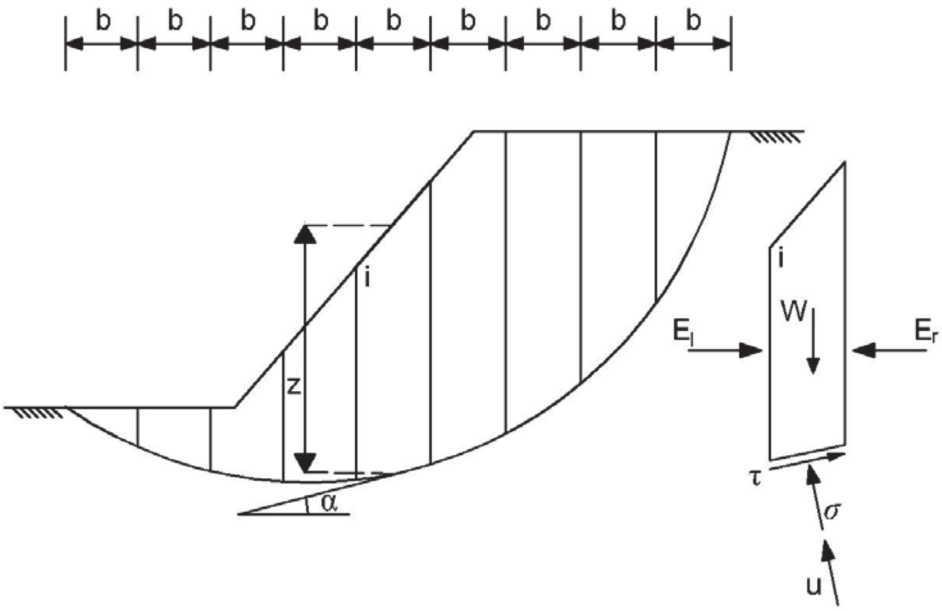

Figure 1. The stresses and forces act on a slice

The safety factor (Fs) of slope is calculated by using Equation 1 in the Bishop method. Because the Fs occurs on both sides of the equation, trial and error method is used to find the Fs. Initially, any value is selected for Fs and then the right side of the equation is calculated. This iterative process continues until the calculated Fs converges to the selected Fs. When the difference of these two values is 0.001 or less, the calculated value is accepted as the safety factor. Determining of the safety factor for different the slope and soil parameters, takes a long time without using any computer program. In this study, Slide computer program is used to determine the safety factor.

$$F_S = \frac{\sum (c'b + (W - u\,b)\tan\phi')\sec\alpha}{\sum W\sin\alpha \left(1 + \frac{\tan\alpha \tan\phi'}{F_S}\right)} \tag{1}$$

Five slope and soil parameters affect the stability of homogenous slopes. These are the weight per unit of volume (γ), the angle of internal friction (Ø), the cohesion (c), the slope height (H) and the angle of slope (m). In this study, the effect of slope and soil parameters on the slope stability has been investigated and has been improved the mathematical model which is used to determine the safety factor.

2 TAGUCHI METHOD

Taguchi Method is a technique, which gives the results of full factorial study with a small number of experiment or study. This technique is a robust and alternative improved method to reduce the cost in the studies of optimization and parametric analysis, to reach results in less time and to determine effects of the parameters on the result. According to Taguchi Method, the parameters affecting the results and the process are divided into two groups, controllable and uncontrollable parameters. The orthogonal array, which depends on the number and the level of parameter, has been used to determine the optimum value of the controllable parameters. In this study, result of full factorial design have been employed instead of the orthogonal arrays. For this study, the selected the parameters and their levels are given in Table 1.

Table 1: Selected parameters and their levels

Level	**Parameters**				
	F_1 γ, kN/m^3	F_2 Ø, (°)	F_3 c, kPa	F_4 H, m	F_5 m
1	14	10	10	5	0,5
2	16	20	30	10	1
3	18	30	50	20	2
4	20	40	70	30	3
5	-	-	-	40	-

In Taguchi Method, effects of the parameters on the result have been determined with the S/N analysis. In the Signal/Noise analysis, S/N ratio is defined as the variation index. If the S/N ratio increases, the product change around the target value decreases. Equation 2 calculates S/N ratio.

$$S/N = -\log_{10}(MSD) \tag{2}$$

The calculation of MSD changes according to the goal of your study. These goals are the larger is better, the smaller the better and the nominal is best, which are given Equations 3-5 respectively.

$$\text{MSD}=\left(\frac{1}{Y_1^2}+\frac{1}{Y_2^2}+...+\frac{1}{Y_n^2}\right)/n \tag{3}$$

$$\text{MSD}=\left(\frac{Y_1^2+Y_2^2+...+Y_n^2}{n}\right) \tag{4}$$

$$\text{MSD}=\left(\frac{(Y_1-Y_0)^2+(Y_2-Y_0)^2+....+(Y_n-Y_0)^2}{n}\right) \tag{5}$$

Here, values of Y1…..Yn are the results of experiment (analysis). n is the number of repetitions for an trial. In Taguchi Method, the expected target value is determined with Equation 6 as.

$$Y_{\text{expected}=}\sqrt{\frac{1}{\text{MSD}_{\text{average}}}} \tag{6}$$

3 RESEARCH FINDINGS

For 1280 different cases, which corresponds to the parameters and their levels (Table 1), the critic safety factors (F_s) have been determined by using Slide computer program. For each trial, the S/N ratios have been calculated by using the Equation 2. The average S/N ratios, which belong to the parameter levels, are given in Table 2.

Table 2: Average S/N ratios

Parameters	**S/N ratio**				
	Level 1	Level 2	Level 3	Level 4	Level 5
γ	4.613	3.956	3.393	2.904	-
Ø	0.583	2.961	4.817	6.507	-
c	-1.333	3.079	5.630	7.492	-
H	10.063	6.013	2.434	0.603	-0.528
m	7.454	4.568	2.008	0.838	-

The change graph, which is drawn by using the average S/N ratios in Table 2, is given in Figure 2. It shows the change between the average S/N ratios and the parameter levels.

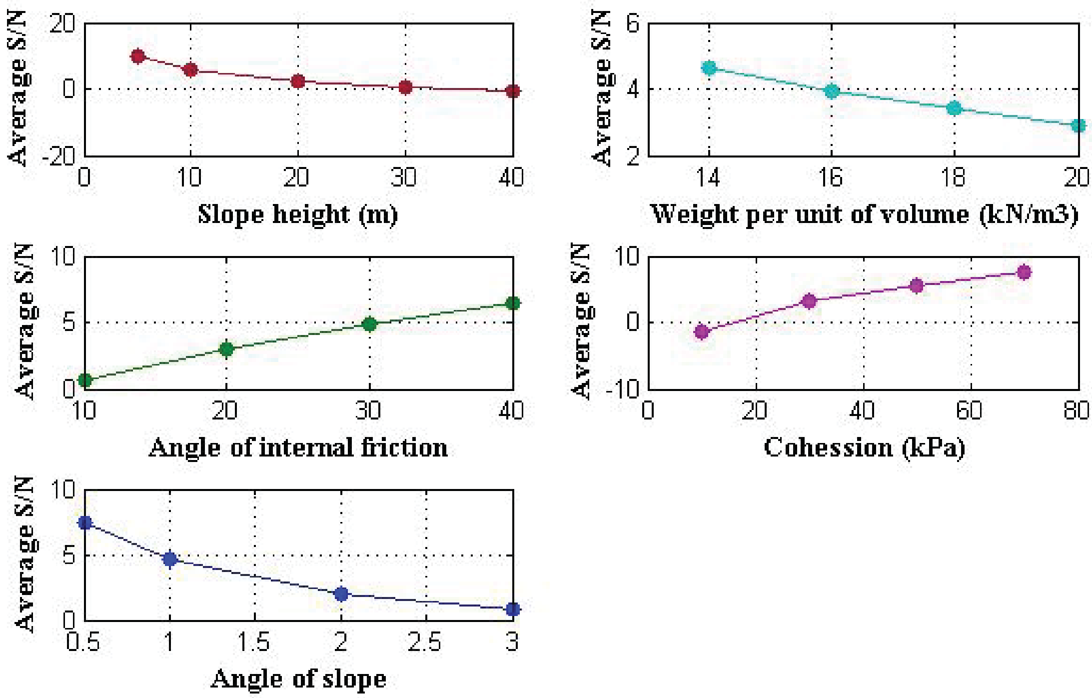

Figure 2. Change between S/N ratio and parameter levels

4 THE DEVELOPMENT OF MATHEMATICAL MODEL

In Taguchi method, studies have been realized according to the average S/N ratios of the parameters. To improve a useful formula, which is used in determining of the safety factor of slope stability, different functions have been selected and then statistical analyses have been performed. End of the studies the best mathematical model is given below;

$$F_s = \sqrt{\frac{1}{10^{-\lambda/10}}} \quad (7)$$

Here, λ is the total effect coefficient and expression of λ is given the Equation 8;

$$\lambda = \psi_\gamma + \psi_\varphi + \psi_c + \psi_H + \psi_m - 14.8676 \quad (8)$$

ψ_γ : the effect coefficient of weight per unit of volume
$\psi_\o$: the effect coefficient of angle of internal friction
ψ_c : the effect coefficient of cohesion
ψ_H : the effect coefficient of slope height
ψ_m : the effect coefficient of angle of slope

The detailed expression of all the effect coefficients are given in Table 3.

Table 3: The Effect Coefficients for safety factors of slope stability

Parameters	Equations
$\gamma_n \leq 20$ kN/m^3	$\psi_\gamma = 0.01\gamma_n^2 - 0.632\gamma_n + 11.53$
Ø≤ 42°	$\psi_\varphi = -0.00087\varphi^2 + 0.2453\varphi + -1.7835$
c_u<70kPa	$\psi_c = 2x10^{-5} c_u^3 - 0.004c_u^2 + 0.3546c_u - 4.602$
H< 19m	$\psi_H = -5.5\ln H + 18.84$
H≥ 19m	$\psi_H = -4.28\ln H + 15.25$
m< 3	$\psi_m = 11.22e^{-0.862m}$

By using improved model for determining of the critic safety factor (Equation 6), the expected safety factors have been calculated for 1280 different cases. According to calculated the relative errors, the values of the expected model safety factors and the determined the Bishop Method safety factors are close to each other. The distribution of the model safety factors and the Bishop safety factors are shown in Figure 3.

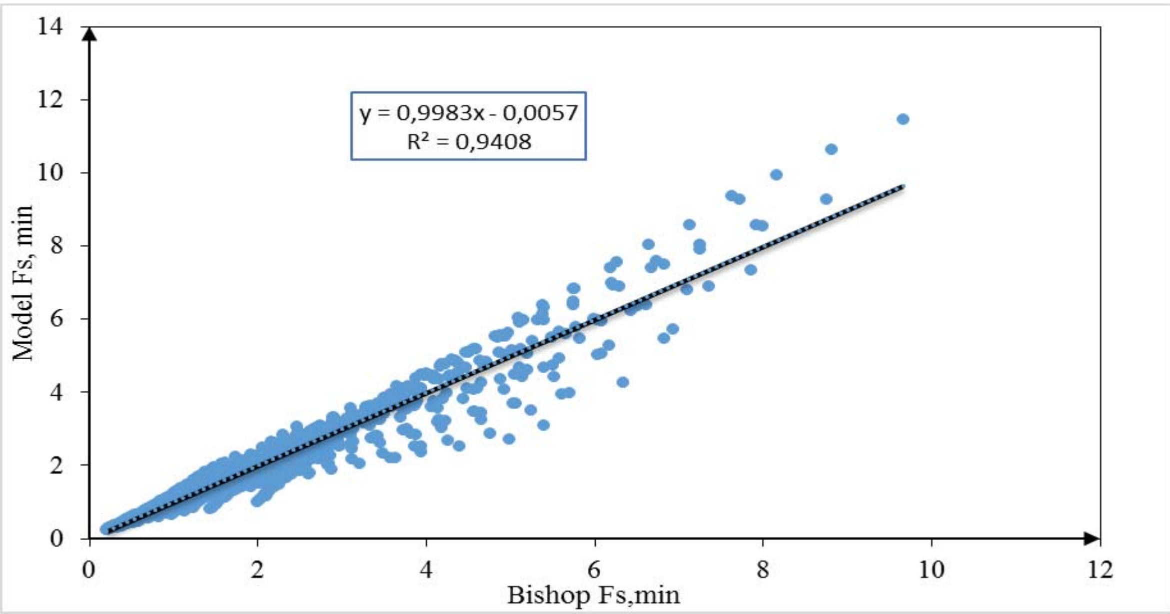

Figure 3. Distribution of the model safety factors and the Bishop safety factors

In Figure 4, from the result of the statistical analysis, the histogram of the relative error-data number (frequency) is shown.

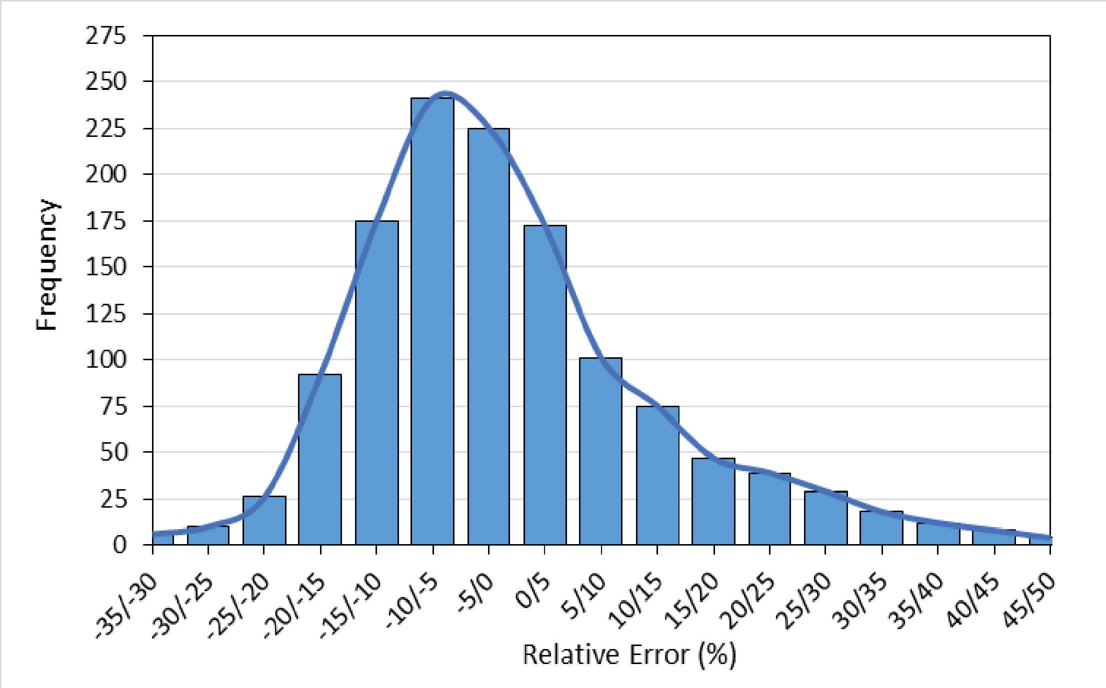

Figure 4. Frequency distribution of relative error

In Table 4, the slope and soil parameters, the safety factors of the model and the Bishop and the values of relative error are given for 12 different cases selected randomly from all analyses.

Table 4: The Trial Results

No	Parameters					Fs(Model)	Fs(Bishop)	Relative Error (%)
	γ, (kN/m³)	Ø, (°)	c (kPa)	H (m)	m			
1	14,00	10,00	10,00	5,00	3,00	0,97	0,88	-10,4
2	19,00	5,00	20,00	7,00	2,00	0,88	0,90	2,5
3	14,00	40,00	50,00	40,00	1,00	2,05	1,91	-7,3
4	16,00	10,00	30,00	5,00	1,00	2,34	2,66	12,0
5	21,00	15,00	60,00	18,00	1,00	1,54	1,53	-0,9
6	16,00	10,00	70,00	20,00	1,00	1,65	1,69	2,6
7	18,00	20,00	30,00	20,00	0,50	1,58	1,68	5,8
8	17,00	35,00	60,00	12,00	0,50	4,78	4,33	-10,5
9	18,00	30,00	10,00	20,00	2,00	0,65	0,68	3,9
10	20,00	20,00	50,00	10,00	3,00	1,49	1,64	9,4
11	15,00	35,00	20,00	18,00	2,00	1,14	1,14	-0,2
12	20,00	40,00	70,00	30,00	2,00	1,74	1,54	-13,2

5 CONCLUSIONS

In this study, the Taguchi Method, which is a robust optimization technique, has been adopted. It is used in the optimization problems and to determine the effects of the parameters on the result. For selected levels, the mathematical model has been improved by using the average S/N ratios belong to 1280 different data. To determine the critic safety factor, the Equation 6 has been improved as the mathematical model. The expected safety factors calculated by the model is quite close to the obtained safety factors determined by the Bishop Method in computer program. This indicates consistency between the expected safety factors and calculated safety factors. In improved model, the average relative error has been determined as 10%. It has been determined that the probability of the absolute errors in expected values being bigger than 20% is 11.8% and being smaller than 10% is 58%. By using the improved model can be determined the critic safety factor with the 10% error without need the computer software. For further parameter and parameter levels, more detailed and reliable models can be improved. The reliability of the results is important in terms of application of Taguchi Method in solution of geotechnical engineering problems.

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